Note: exam room change:
Exams 2,3 take place in STO B50

Lab 5 is in SCI 134

Please, login into webassing, locate LectureMCQ_L11 (PY105) and answer question 1 (but ONLY Q1!).

Pleas sign in using the sign-in sheets on the bench. Thank you

Good morning!

Impulse and Momentum

average

$$\overline{F}_{net} = m\overline{a} = m\frac{\Delta V}{\Delta t} = m\frac{V_2 - V_1}{\Delta t} \quad \text{hence} \quad \boxed{\overline{F}_{net}\Delta t = m\mathbf{v}_2 - m\mathbf{v}_1}$$

N2L

BTW: this is how Newton wrote it

<u>Two new physical quantities:</u>

Impulse of the force $\boldsymbol{J} = \overline{\boldsymbol{F}}_{net}\Delta t$	Linear Momentum of the object $\boldsymbol{P} = m\boldsymbol{v}$

The unit is the same: N s = kg*m/s

Relationships for impulse and momentum:

$$\overline{\boldsymbol{J}} = \overline{\boldsymbol{F}}_{net}\Delta t$$

$$\boldsymbol{P} = m\boldsymbol{v}$$

$$\boxed{\vec{F}_{Net\ Ave} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}} \longleftrightarrow \overline{\boldsymbol{F}}_{net}\Delta t = \boldsymbol{P}_2 - \boldsymbol{P}_1 \longleftrightarrow \boxed{\overline{\vec{J}} = \Delta\vec{p}} \leftarrow$$

IMT = 2NL !

Impulse – Momentum Theorem (IMT) "works" for ***any*** system

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

If $J \neq 0$

$$\vec{p}_2 = \vec{p}_1 + \vec{J}$$

If $J = 0$

$$\vec{p}_2 = \vec{p}_1$$

$$\vec{J} = \vec{F}_{Net\,Ext\,Ave}\Delta t \rightarrow \vec{F}_{Net\,Ext\,Ave} = \frac{\vec{J}}{\Delta t}$$

F = const

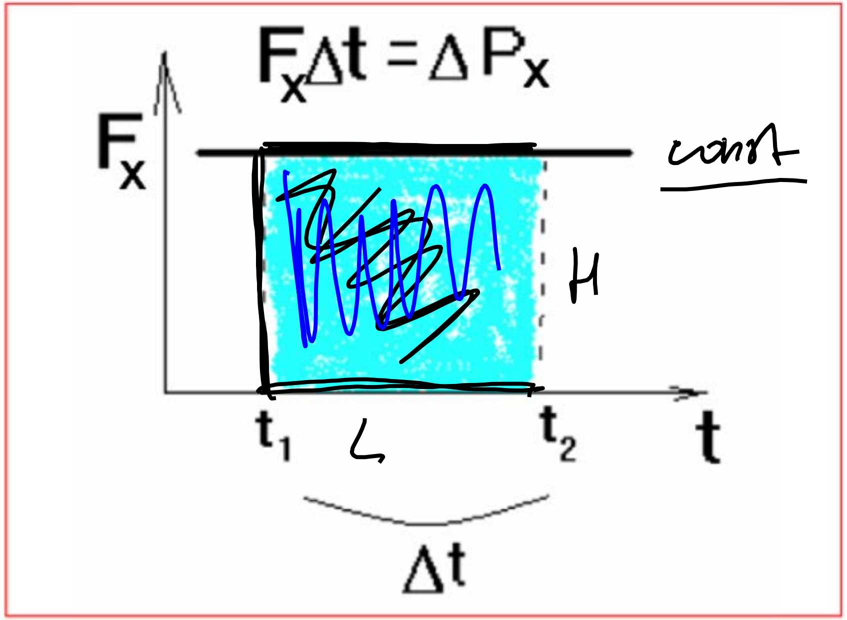
1-D case

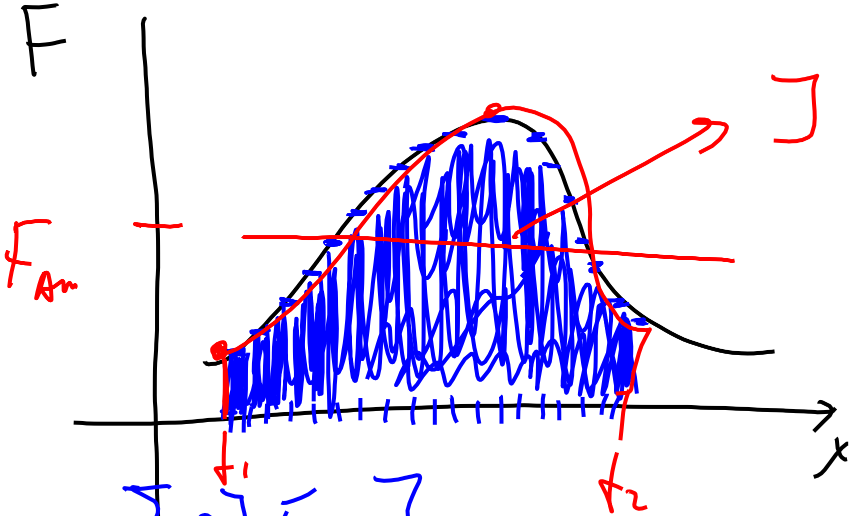
$$\vec{J} = \vec{F}_{Net\,Ext\,Ave}\Delta t$$

$$F_{xav} = \frac{J}{\Delta t}$$

$$\vec{p}_2 = \vec{p}_1 + \vec{J}$$

$F \neq$ const => ??



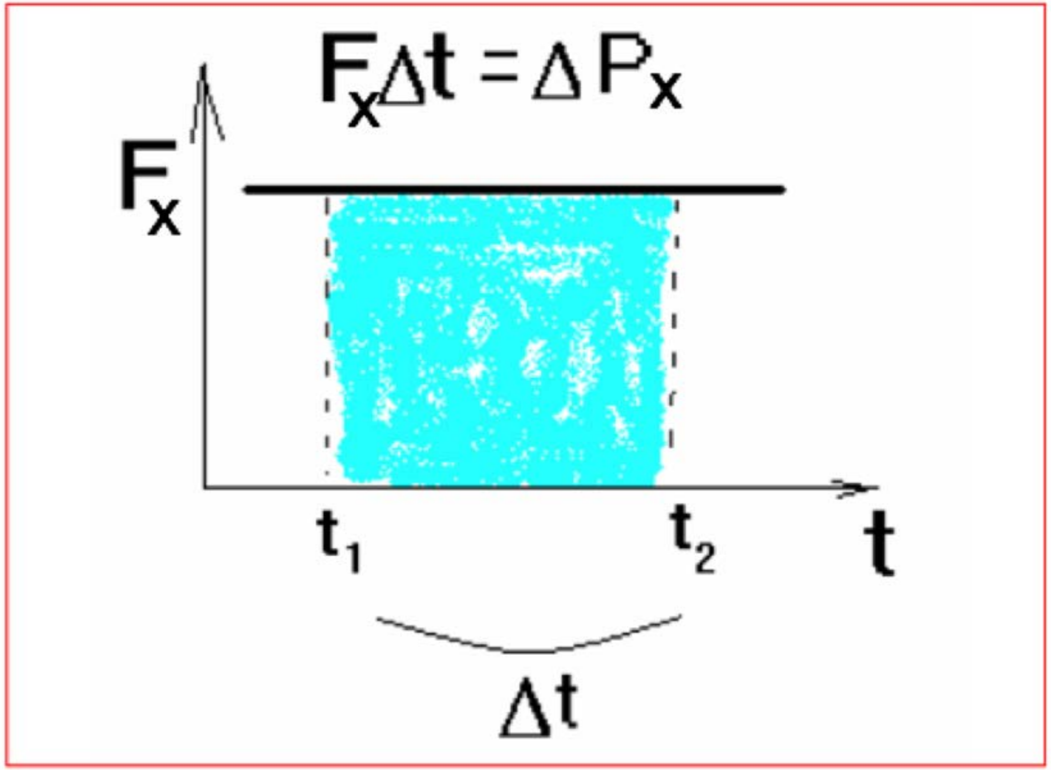


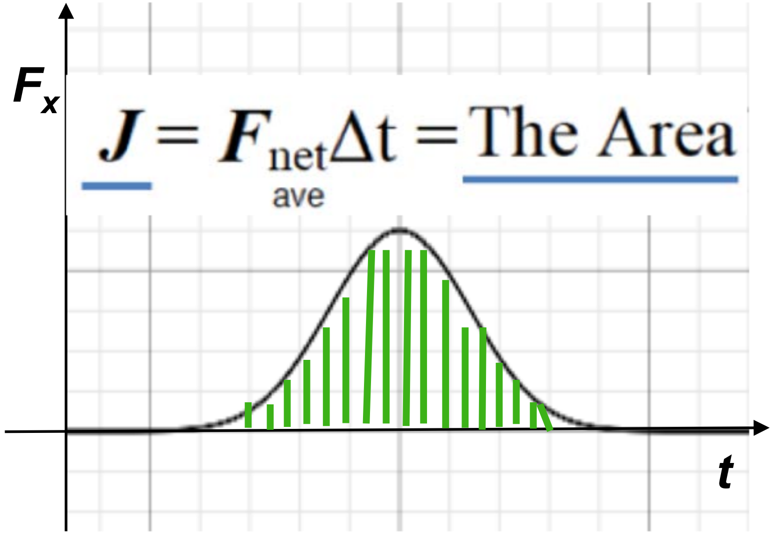
$$J = F \cdot \Delta t$$

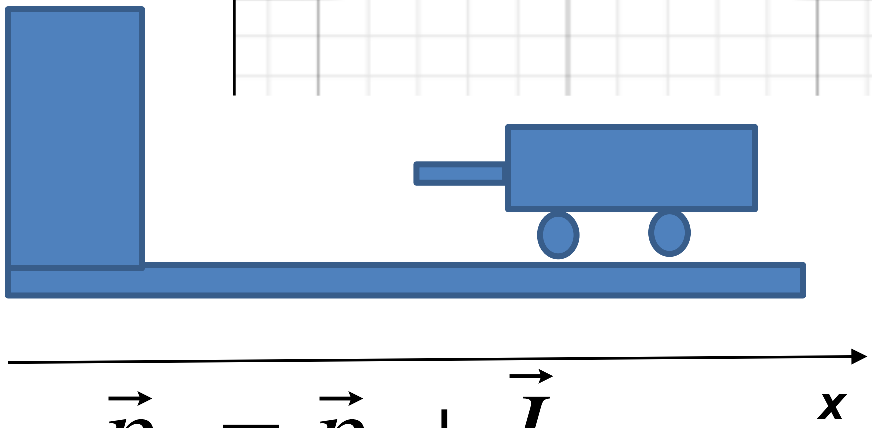
$$\sum \Delta J = J$$

$$\vec{J} = \vec{F}_{Net\,Ext\,Ave}\,\Delta t$$

1-D case



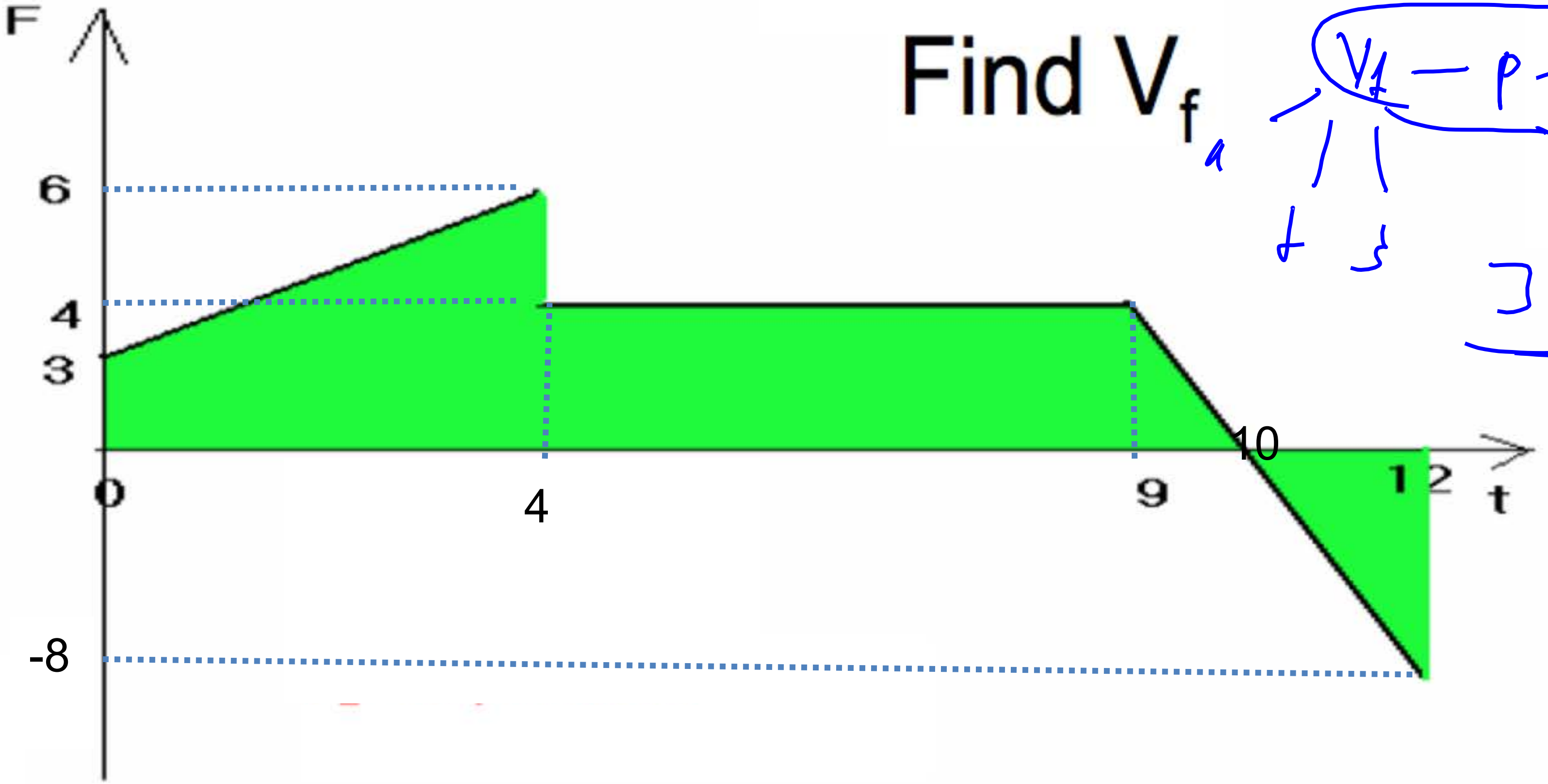




$$\vec{p}_2 = \vec{p}_1 + \vec{J}$$

$$\underline{J} = F_{\text{net ave}}\,\Delta t = \underline{\text{The Area}} \quad \Rightarrow \quad \underline{\Delta P = \text{The Area}}$$

Find V_f

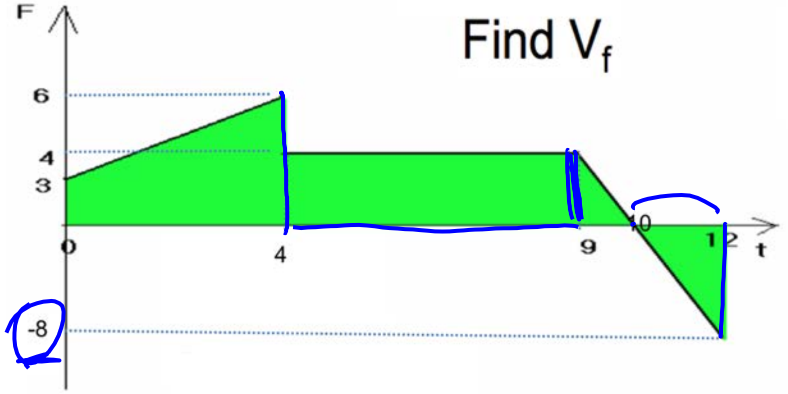


$V_f - P - J - A$

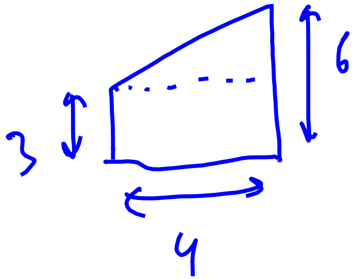
$J = \Delta P$

The graph represents a force acing on a 3 kg box initially traveling to the left at the speed of 4 m/s. Find the final velocity of the box.

Find V_f



The graph represents a force acing on a 3 kg box initially traveling to the left at the speed of 4 m/s. Find the final velocity of the box.



$J = A_{rea} = A_1 + A_2 + A_3 + A_4$

$A_1 = \frac{1}{2}(3+6)\cdot 4$

$A_2 = (9-4)\cdot 4$

$A_3 = \frac{1}{2}(10-9)\cdot 4$

$A_4 = -\frac{1}{2}(12-10)\cdot 8 = \frac{1}{2}(12-10)\cdot(-8)$

$J = A_1 + A_2 + A_3 + A_4 = P_f - P_i$

$V_f = \frac{P_f}{m} = \frac{P_i + J}{m} = \frac{m\cdot(-4) + J}{m} = -4 + \frac{J}{3};$

A "happy" ball knocks a block down, a "sad" ball" does not.

Which ball exerts larger force on the block?

A) Happy

B) Sad

C) Forces are equal

D) Impossible to tell

Webassign: L11 Q2

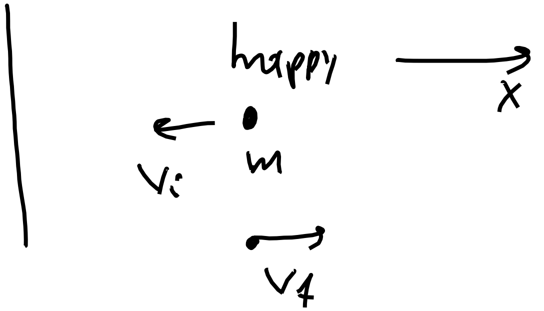
Which ball exerts larger force on the block?

A) Happy

B) Sad

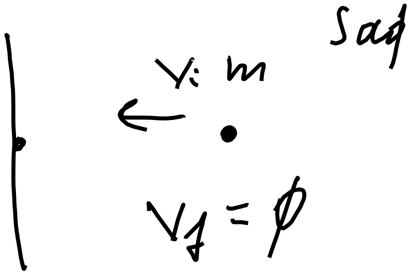
C) Forces are equal

D) Impossible to tell



$|X_i| = |V_f| = v$

$$F = \frac{\Delta P}{\Delta t} = \frac{P_f - P_i}{\Delta t} = \frac{mV_f - (-m|V_i|)}{\Delta t} = \frac{mv - -mv}{\Delta t} = 2 \cdot \frac{mv}{\Delta t}$$



$$F = \frac{mV_f - (-m|V_i|)}{\Delta t} = \frac{0 + mv}{\Delta t} = \frac{mv}{\Delta t}$$

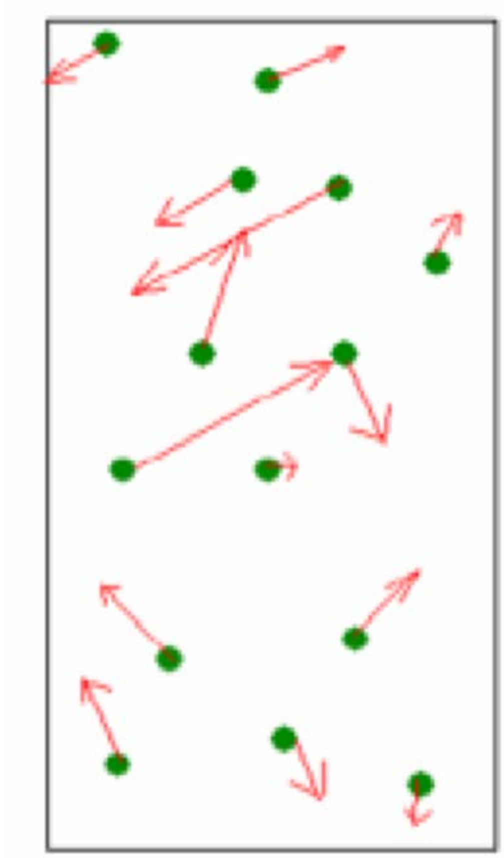
Which ball exerts larger force on the block?

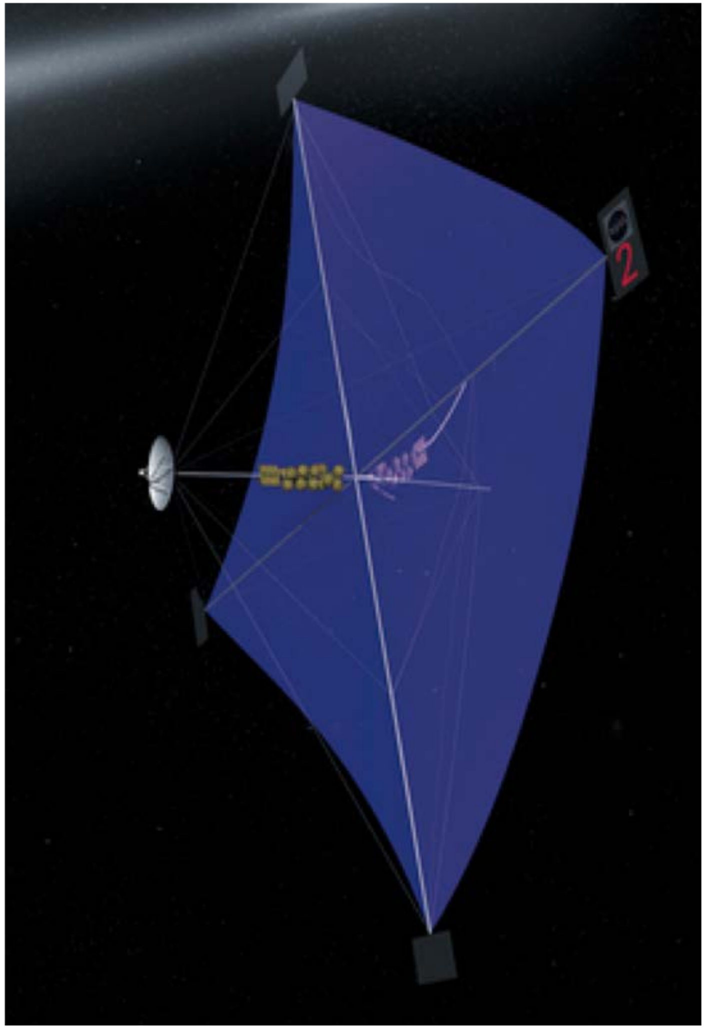
Happy Ball	Sad Ball
$F_h \Delta t_h = P_{h\,2} - P_{h\,1}$	$F_s \Delta t_s = P_{s\,2} - P_{s1}$
$F_h \Delta t_h = -\mathbf{P} - \mathbf{P} = -2\mathbf{P}$	$F_s \Delta t_s = 0 - \mathbf{P} = -\mathbf{P}$

$$|F_h| > |F_s|$$

Happy!

Applications!





Impulse – Momentum Theorem (IMT)

works for <u>***any***</u> system:

If (1) $\vec{F}_{net} = 0$ or (2) $\Delta t \to 0$ => $\vec{J} = 0$!

A collision

A <u>very</u> SHORT contact time

$$\vec{F}_{net}\,\Delta t = \vec{P}_f - \vec{P}_i$$

SYSTEM SYSTEM SYSTEM

0 => $0 = \vec{P}_f - \vec{P}_i$

SYSTEM

=> $\vec{P}_f = \vec{P}_i$

Law of conservation of linear momentum! (***LCLM***):

when there are **NO *outside*** forces acting on the system, **or** they are canceled out (<u>**net** *external* force equals zero</u>), **or** the interaction takes almost no time.

SYSTEM

$$\vec{P}_f = \vec{P}_i$$

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$ **LCLM**

$$\vec{p}_{System} = const$$ **LCLM**

KE_{1i} KE_{2i}

Collisions

Based on ΔKE

1. Absolutely inelastic (AIC): **Form one object** ("stick together").

Inelastic ($W_{fr} < 0$) Regular case

Super-elastic ($ME_2 > ME_1$)

2. (Ab) Elastic: <u>**Mechanical energy**</u> **of the system is** <u>***conserved***</u>.

An explosion = SupEC = an AIC "back in time"

$$\vec{p} = m\vec{v}$$

1 or 2

i or *f*

ALWAYS! FOR *ALL* collisions!

LCLM $$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$

After the collision

Before the collision

$$\Delta KE = W_{net}$$

1-D

x

$$m_1 v_{x1i} + m_2 v_{x2i} = m_1 v_{x1f} + m_2 v_{x2f}$$

Each V_x represents the x- component of a vector!!!

Special Case of *Absolutely Inelastic Collision* (AIC)

Same masses, same speeds.

Webassign: L11 Q3

Is ***mechanical energy*** conserved in this collision?

1) Yes
2) No
3) Impossible to answer

Is *energy* conserved in this collision?

1) Yes
2) No
3) Impossible to answer

<u>Special</u> Case of *Absolutely Inelastic Collision* (AIC)

Same masses, same speeds.

Is *<u>mechanical</u> energy* conserved in this collision?

1) Yes
2) No
3) Impossible to answer

$ME = KE + PE$ → Law or definition

$ME_i \; > \; = \; < \; ME_f$

$ME_i = KE_{1i} + \cancel{PE_{1i}} + KE_{2i} + \cancel{PE_{2i}} = \frac{mv^2}{2} + \frac{mv^2}{2} = mv^2$

$ME_f = 0$

<u>Special</u> Case of *Absolutely Inelastic Collision* (AIC)

Same masses, same speeds.

Is *mechanical energy* conserved in this collision?

1) Yes
2) No
3) Impossible to answer

Is *energy* conserved in this collision?

Always

AIC

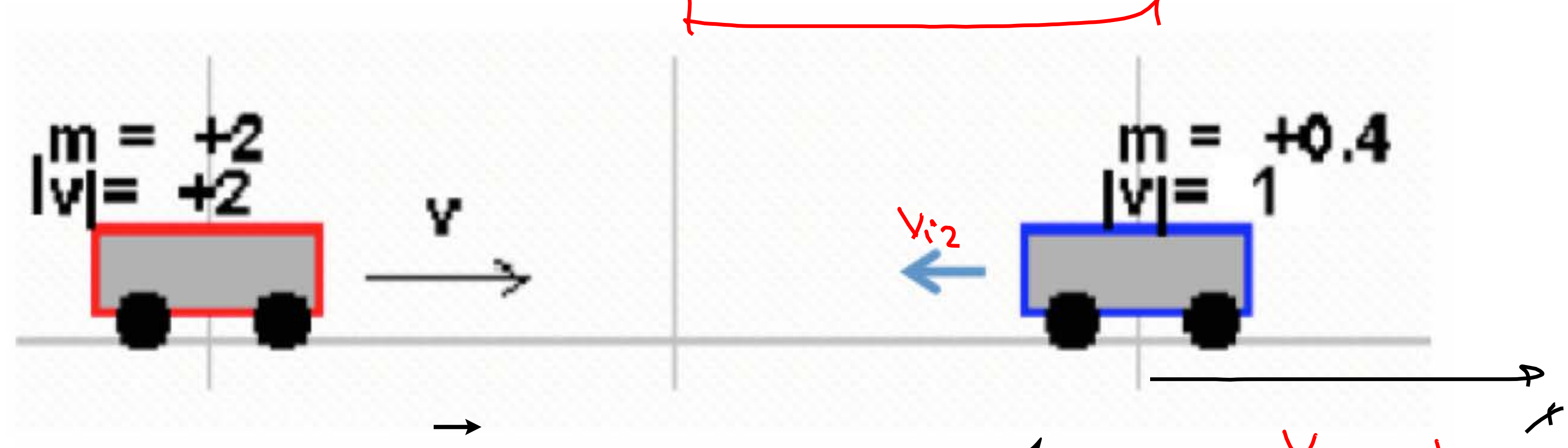
Two carts of different mass and velocity (see the picture) collide and got stuck to each other.

Find the velocity of the carts after the collision.

$$\vec{J} = \vec{F}_{Net\,Ave}\Delta t$$

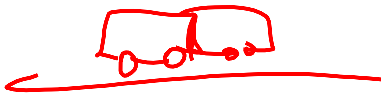
$$\vec{F}_{Net\,System} = 0 \quad \Rightarrow \quad \vec{p}_{System} = const$$

Assume the collision is *absolutely inelastic*



$$\vec{p}_{System} = const$$

$$v_{1fx} = v_{2fx} = u$$



$$\vec{p}_{si} = \vec{p}_{sf}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1 \cdot v_{1ix} + m_2 \cdot v_{2ix} = m_1 \cdot v_{1fx} + m_2 \cdot v_{2fx}$$

$$2 \cdot 2 + 0.4 \cdot (-1) = 2 \cdot v_{1fx} + 0.4 \cdot v_{2fx}$$

$$4 - 0.4 = 2 \cdot u + 0.4 \cdot u$$

$$\frac{3.6}{2.4} = u$$

Two objects hit each other.

We call it *a collision!*

For *any* collision we ALWAYS can write the *law of conservation of linear momentum.* $P_{\text{system-before-collision}} = P_{\text{after-before-collision}}$ or

$$m_1\mathbf{v}_{1bc} + m_2\mathbf{v}_{2bc} = m_1\mathbf{v}_{1ac} + m_2\mathbf{v}_{2ac}$$

Now, with taking into a consideration the directions of the velocities, we can write (and the final velocities are the same!)

$$2*2 + 0.4*(-1) = 2\mathbf{v}_{ac} + 0.4\mathbf{v}_{ac} = 2.4\ \mathbf{v}_{ac}$$

That gives us $v_{ac} = 3.6 / 2.4 = 1.5$ m/s

Cont.

AIC

Two carts of different mass and velocity (see the picture) collide and got stuck to each other.

Find the velocity of the carts after the collision. **=> done**

Find the loss of the kinetic energy (how much energy got transferred into a heat because of friction?)

$$KE_{1i} = \frac{2 \cdot 2^2}{2} \qquad KE_{2i} = \frac{0.4 \cdot 1^2}{2}; \qquad KE_f = \frac{(2+.4) \cdot 1.5^2}{2}$$

Cont.

AIC

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That gives us $v_{ac} = 3.6/2.4 = 1.5$ m/s

The loss of the kinetic energy (which is equal to the work of friction) can be found as

$$\Delta K = W_{fr} = \tfrac{1}{2}(m_1 + m_2)v_{ac}^2 - \tfrac{1}{2}m_1 v_{1bc}^2 - \tfrac{1}{2}m_2\mathbf{v}_{2bc}^2 \quad = -1.5\ \text{J}$$

An explosion, a separation = a AI collision “backwards”

Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

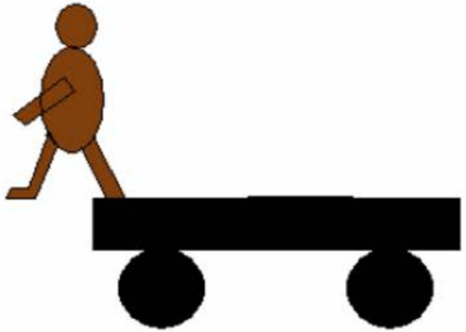
One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

The completely similar solution you would have to use for solving the problems like the following:

<u>The gun fires a bullet</u>

or

<u>A man (dog) jumps of the cart (or boat)</u>



or

<u>A grenade explodes into two parts</u>

Newton's Third Law

When one object exerts a force on a second object, the second object exerts a force of equal magnitude, in the opposite direction, on the first object.

Note that this law relates forces that are acting on different objects belonged to <u>different</u> free body diagrams.

vs.

General Case of *Elastic* Collision (a.k.a. AEC)

x

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$ **LCLM** ***and***

$$ME_{1f} + ME_{2f} = ME_{1i} + ME_{2i}$$

1-D; For a horizontal surface

LCME
(no friction)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Each v represents an x- component of a vector!!!

$$KE_{1f} + KE_{2f} = KE_{1i} + KE_{2i}$$

No friction ; $ME = const$

x

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2}$$

→ Physics is done!

Know: $v_1 ; v_2$

$u_1\ u_2 - ?$

$$u_1 = \frac{m_1 v_1 + m_2 v_2 - m_2 u_2}{m_1}$$

$$a^2 - b^2 = (a-b)\cdot(a+b)$$

$$m_1 v_1^2 - m_1 u_1^2 = m_2 u_2^2 - m_2 v_2^2$$

$$m_1 (v_1^2 - u_1^2) = m_2 (u_2^2 - v_2^2)$$

$$m_1 \cdot \cancel{(v_1 - u_1)} \cdot (v_1 + u_1) = m_2 \cdot \cancel{(u_2 - v_2)} \cdot (u_2 + v_2)$$

$$m_1 v_1 - m_1 u_1 = m_2 u_1 - m_2 v_2$$

$$m_1 (v_1 - u_1) = m_2 (u_1 - v_2)$$

$$v_1 + u_1 = v_2 + u_2$$

1) $m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 u_2$ and 2) $\frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} = \frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2}$

$\Rightarrow V_1 + u_1 = u_2 + V_2$; WORK:

From 1: $m_1 V_1 - m_1 u_1 = m_2 u_2 - m_2 V_2$ or $m_1 (V_1 - u_1) = m_2 (u_2 - V_2)$

Fan 2: $m_1 V_1^2 - m_1 u_1^2 = m_2 u_2^2 - m_2 V_2^2$

or

$m_1 (V_1^2 - u_1^2) = m_2 (u_2^2 - V_2^2)$

use: $a^2 - b^2 = (a-b)(a+b)$

$m_1 (V_1 - u_1)(V_1 + u_1) = m_2 (u_2 - V_2)(u_2 + V_2)$

is equal to because of

$\Downarrow$

$V_1 + u_1 = u_2 + V_2$

<u>General</u> Case of *Elastic* Collision (a.k.a. AEC)

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$

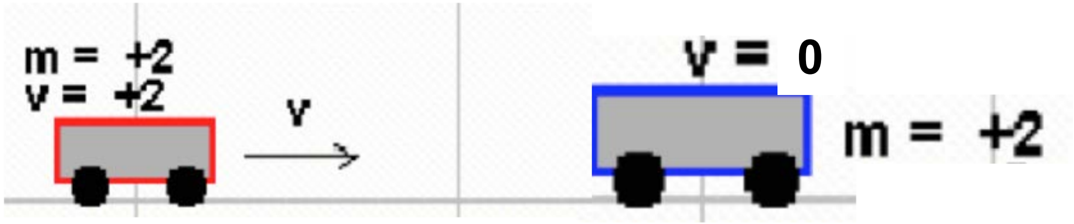
and

$$ME_{1f} + ME_{2f} = ME_{1i} + ME_{2i}$$

For a horizontal surface

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \qquad v_1 + u_1 = u_2 + v_2$$

<u>Each</u> u <u>and</u> v <u>represents an x- component of a vector!!!</u>



Special case

Find the velocity of the carts after the collision.

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$$v_1 + u_1 = u_2 + v_2$$

$$\begin{cases} \cancel{2}\cdot 2 + \cancel{2\cdot 0} = \cancel{2}\cdot u_1 + \cancel{2}\cdot u_2 \\ 2 + u_1 = u_2 + 0 \end{cases}$$

Physics is done!

$$\Rightarrow$$

$$2 = u_1 + u_2$$

$$2 + u_1 = u_2$$

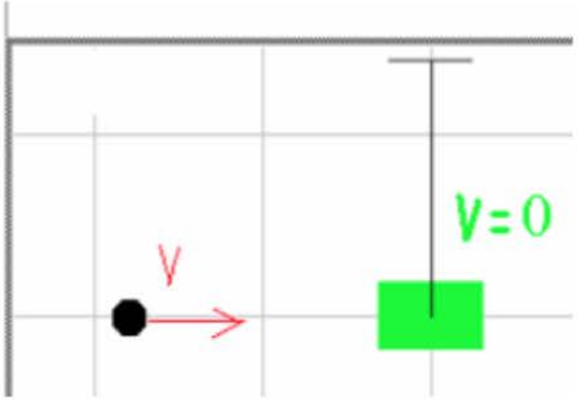
$$2 = u_1 + 2 + u_1$$

$$2 - 2 = 2\cdot u_1$$

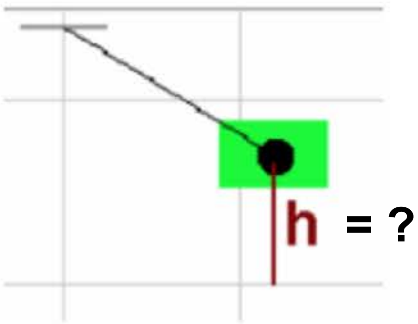
$$0 = u_1$$

Ballistic Pendulum

A 10 gram bullet traveling at 100 m/s hits resting 990 gram pendulum and stuck in it.



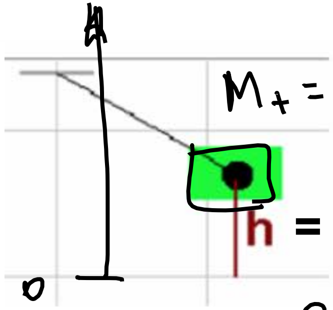
Find the maximum height of the pen-dulum with the bullet



Ballistic Pendulum

A 10 gram bullet traveling at 100 m/s hits resting 990 gram pendulum and stuck in it.

Find the maximum height of the pendulum with the bullet



$M_t = .99 + .01 = 1\,kg$

$M = .99\,kg$

$m = 0.01\,kg$

$V_i = 100\,m/s$

$PE_f = mgy =$

$= 1 \cdot 10 \cdot h$

LMCQ11 Q4

1. Yes, it is correct
2. No, it is wrong

$KE_i = \frac{0.01 \cdot 100^2}{2} = 50\,J$

$50 = 10 \cdot h$

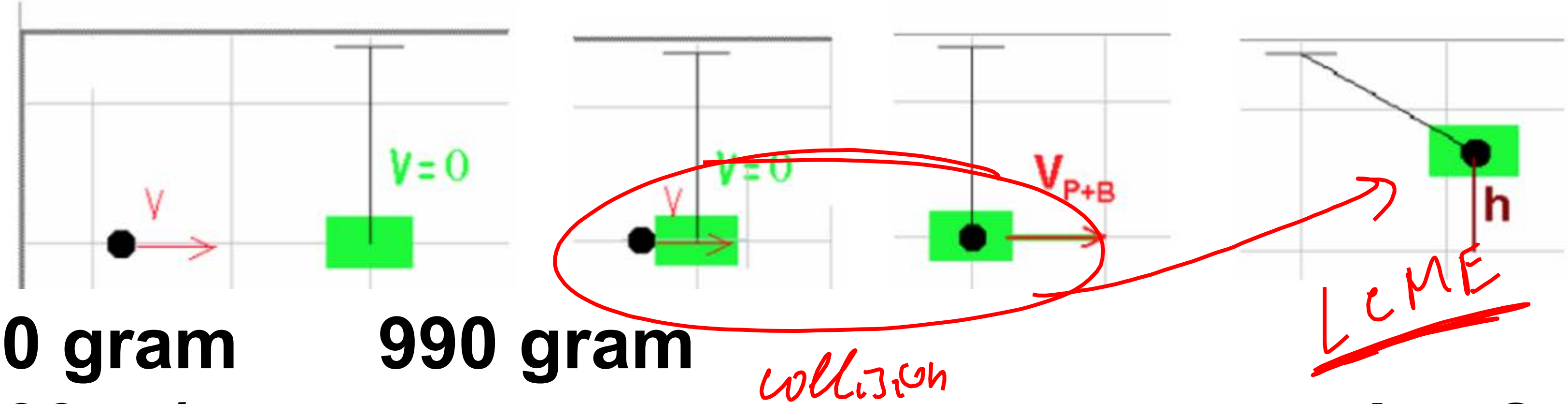
$h = \frac{50}{10} = 5\,m$

$KE_i = PE_f$; $KE_i + W_k = PE_f$

wrong

Ballistic Pendulum

Start	Before the collision	After the collision	End

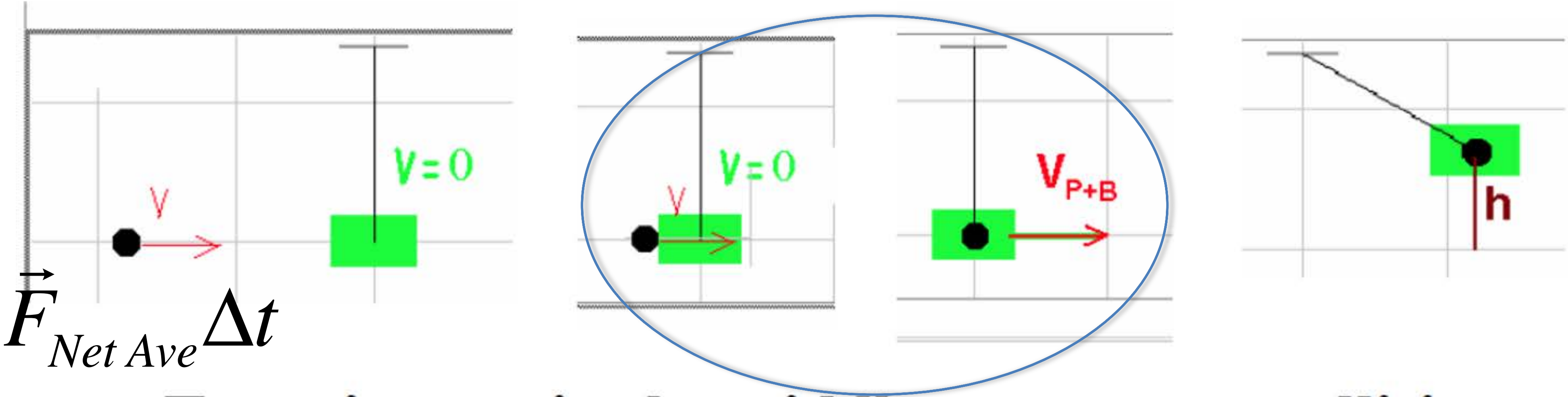


10 gram **990 gram**
100 m/s **at rest** **=> h = ?**

How many different "mini-problems" are in this one problem?

Ballistic Pendulum

Start | Before the collision | After the collision | End



$$\vec{J} = \vec{F}_{Net\ Ave}\Delta t$$

Two pictures in the middle represent a collision

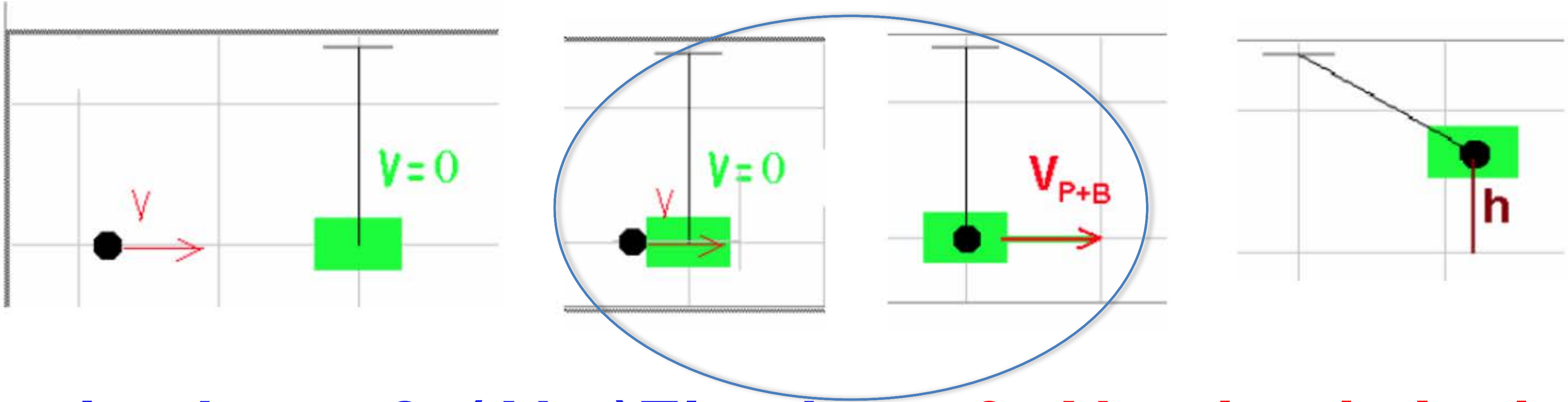
$$\Delta t \approx 0$$

$$\vec{p}_i = \vec{p}_f$$

What kind of a collision is it?

Ballistic Pendulum

Start | Before the collision | After the collision | End



1. Inelastic
2. (Abs)Elastic
3. Absolutely Inelastic
4. Super-elastic

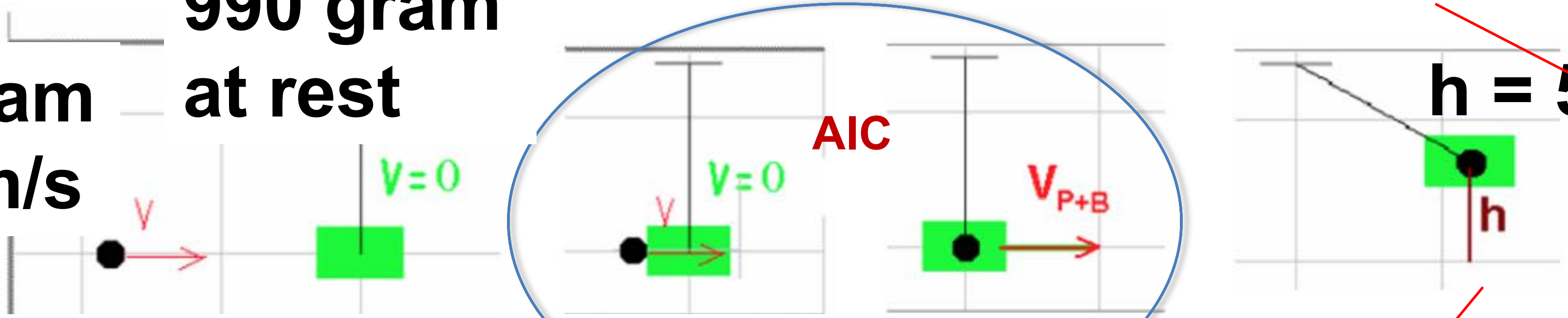
Ballistic Pendulum

Start	Before the collision	After the collision	End

10 gram
100 m/s

990 gram
at rest

h = 5 m ??



LMCQ12 Q3

1. ~~Yes, it is correct~~
2. No, it is wrong

Needs $W_{friction} \neq 0$

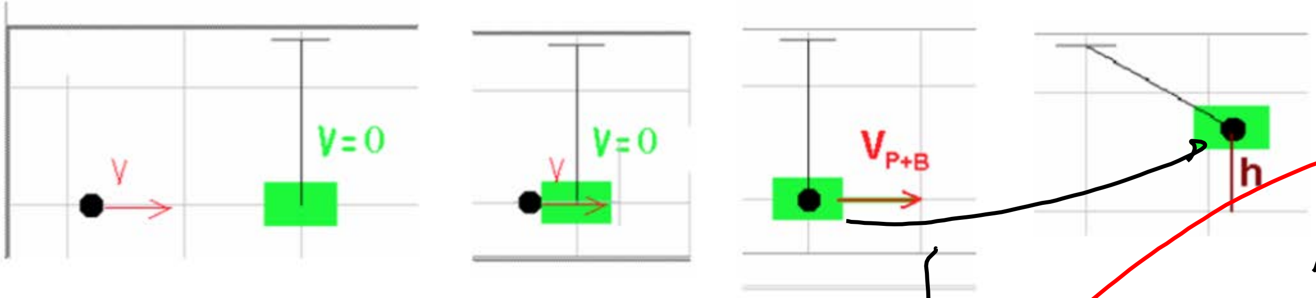
$$\frac{mV^2}{2} + \emptyset = (m+M)gh$$

$$h = \frac{mV^2}{2g(M+m)} = 5 \text{ m}$$

Ballistic Pendulum

Start	Before the collision	After the collision	End

Y



$$m_B \cdot V + \phi = (m_B + M_p)\cdot u$$

LCLM

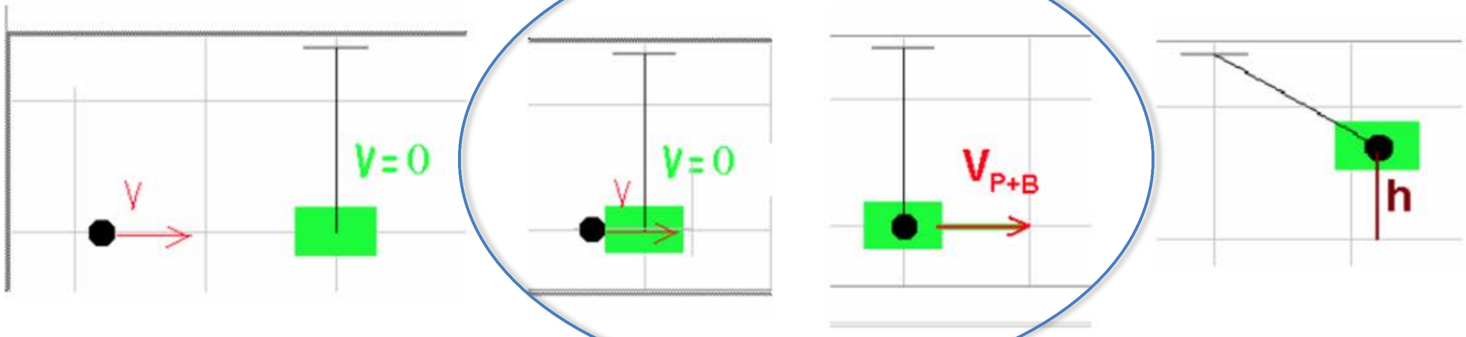
$$\frac{(m_B + M_p)\,u^2}{2} = (m_B + M_p)\cdot g \cdot h$$

LCME

Physics is done

Ballistic Pendulum

Start	Before the collision	After the collision	End



Two pictures in the middle represent an *absolutely inelastic collision*

$m_B V_B = (m_B + M_P) V_{P+B}$ (Conservation of Momentum)

***During* the collision the mechanical energy of the system is *not* conserved!**

But *after* it, E is conserved!

$$\frac{(m_B + M_P) V_{P+B}^{\;2}}{2} = (m_B + M_P) g h$$

Ballistic Pendulum

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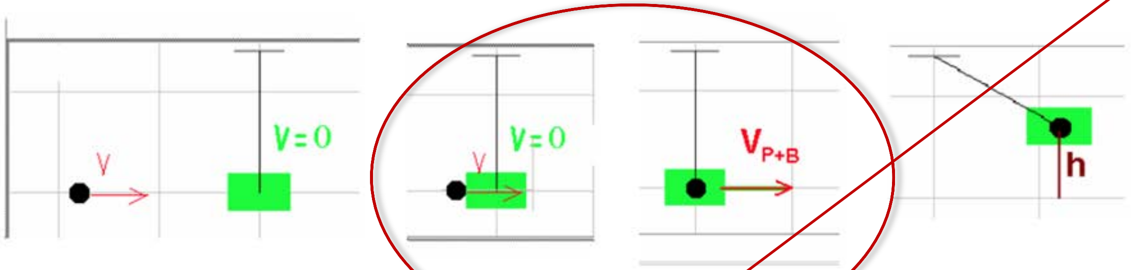

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$0.01 * 100 = 1 * v$

$v = 1$ m/s

$$h = \frac{v^2}{2g}$$

$$h = \frac{1^2}{20}$$

h = 0.05 m = 5 cm

NOT 5 m!!

Ballistic Pendulum

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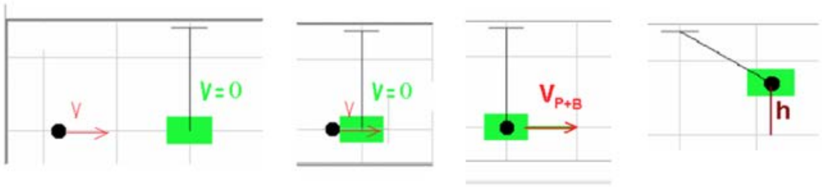

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=

Shooting a piece of wood

The two-part process: (1) collision (2) energy change

1) LCLM + 2) LCME

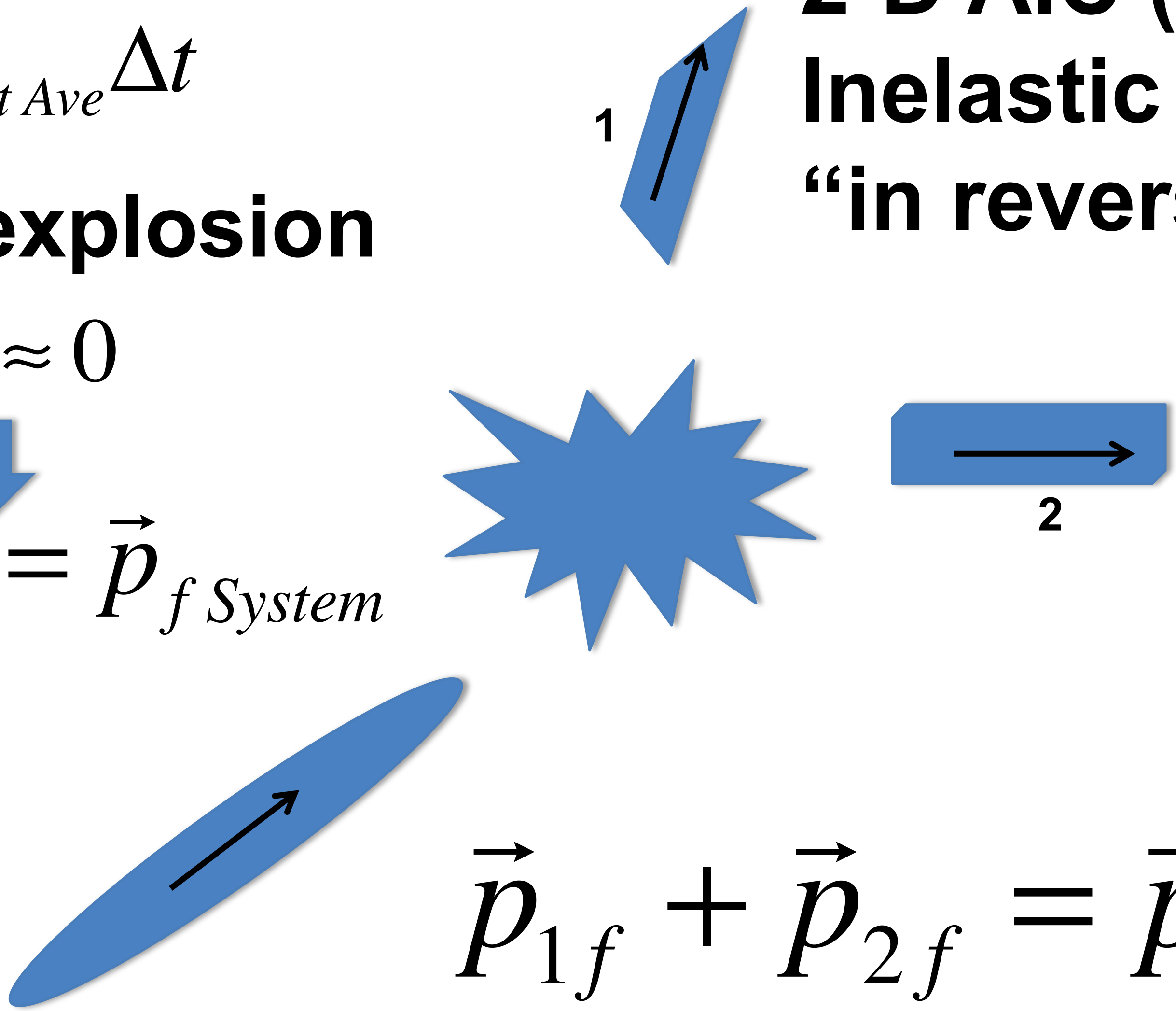
$$\vec{J} = \vec{F}_{Net\,Ave}\Delta t$$

2-D AIC (Absolutely Inelastic Collision "in reverse")

or an explosion

$$\Delta t \approx 0$$

$$\vec{p}_{i\,System} = \vec{p}_{f\,System}$$



$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$

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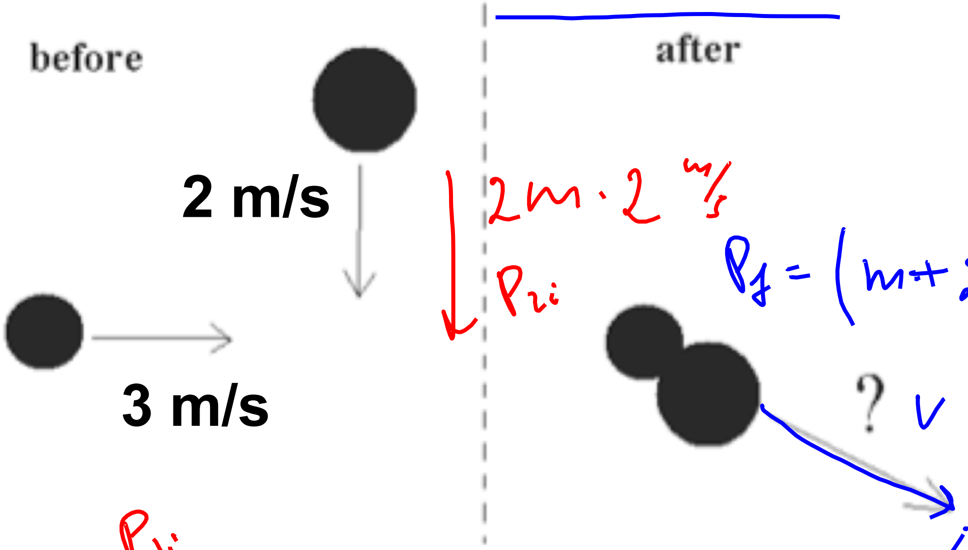
M = 2 m

Initial velocities are at 90^0.



$2m \cdot 2\ m/s$

P_{2i}

P_{1i}

$m \cdot 3\ \frac{m}{s}$

$\tan\theta = \frac{P_{2i}}{P_{1i}}$

$P_f = (m + 2m) \cdot v = 3m \cdot v$

P_{1i} + P_{2i} = P_{ti} =

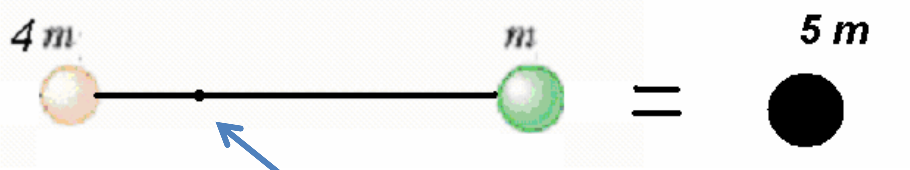
$P_{ft} = \sqrt{P_{1i}^2 + P_{2i}^2} = \; = P_{t4}$

$= \sqrt{(m \cdot 3)^2 + (2m \cdot 2)^2} = \sqrt{m^2 \cdot 9 + m^2 \cdot 16} =$

$= m \cdot \sqrt{25} = m \cdot 5 = 3m \cdot v \Rightarrow v = \frac{5}{3}\ m/s$

<u>The meaning of CM</u> or CofM: Center of Mass

The center-of-mass is the point that moves as though *all the mass of the system* is concentrated there.



(look at the system from afar!)

If we do not need to know the behavior of the individual parts of the system we can replace the system with a point-like object of the mass of the system and apply the Newton's II Law to it.

$$\underbrace{\sum \vec{F}}_{\text{all forces on the system}} = M_{system} \bullet \vec{a}_{CM}$$

(= all external forces on the system)