No labs today

Please, login into webassing, locate LectureMCQ_L12 (PY105) and answer question 1 (**but ONLY Q1!**).

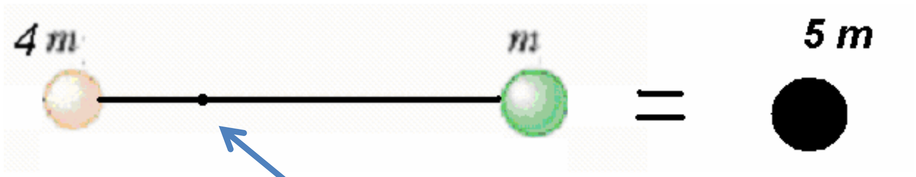
Pleas sign in using the sign-in sheets on the bench. Thank you

Note: exam room change:
Exams 2, 3 take place in STO B50

Good morning!

<u>The meaning of CM</u> or CofM: Center of Mass

The center-of-mass is the point that moves as though *all the mass of the system* is concentrated there.



(look at the system from afar!)

If we do not need to know the behavior of the individual parts of the system we can replace the system with a point-like object of the mass of the system and apply the Newton's II Law to it

$$\sum_{\text{all forces on the system}} \vec{F} = M_{system} \bullet \vec{a}_{CM}$$

(= all external forces on the system)

The meaning of CM

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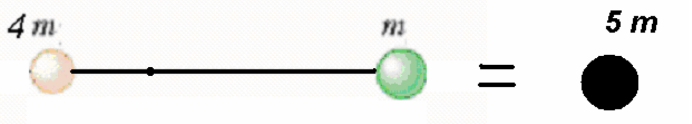

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$$\sum_{\text{all forces on the system}} \vec{F} = M_{system} \bullet \vec{a}_{CM}$$

(= all external forces on the system)

The old name for CofM is Center of Gravity (CG)

IF $\sum \vec{F} = 0 \Rightarrow \vec{a}_{CM} = 0$

$\vec{V}_{CM} = \text{const}$!

$\vec{P}_{system} = \vec{V}_{CM} * M = \text{const}$!

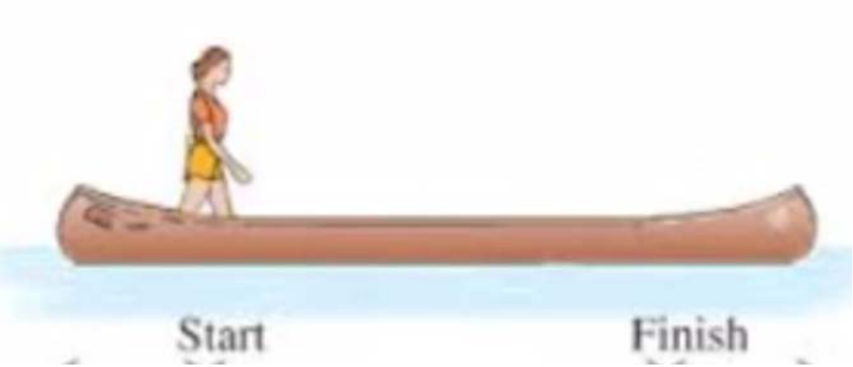
LCLM !

IF $\sum \vec{F} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow \vec{V}_{CM} = const$!

In particular, if CofM was at ***rest*** and there are ***NO external forces*** acting on the system, CofM ***remains at rest*** even if the parts of the system are moving relative to each other.

https://www.youtube.com/watch?v=mM2V7sfAdB8

http://physicstasks.eu/1148/walking-on-the-boat



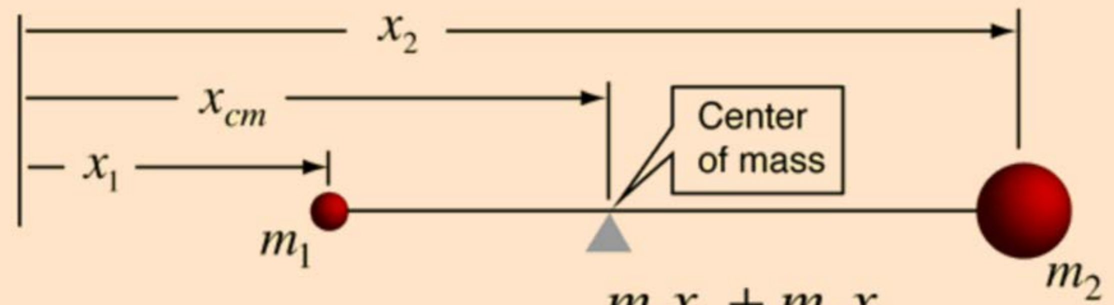
$$\sum \vec{F} = M_{system} \bullet \vec{a}_{CM}$$

all forces on the system

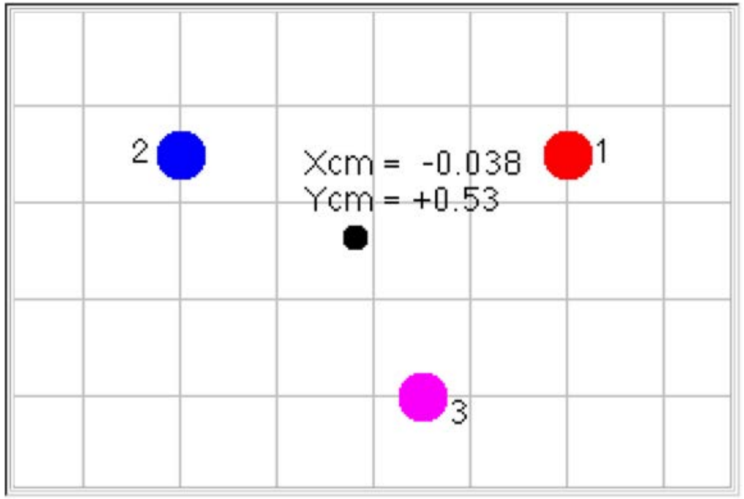
(= all external forces on the system)

Center of Mass equations

=> ... ...



For two masses: $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

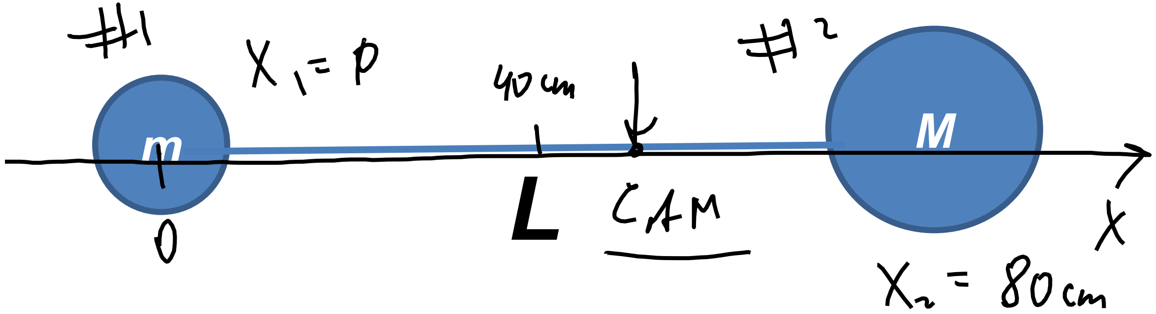


$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

http://www.batesville.k12.in.us/physics/APPhyNet/Dynamics/Center%20of%20Mass/2D_1.html

Find the location of the CofM



$M = 600$ g, $m = 250$ g, $L = 80$ cm

Webassign: L12 Q2

The CofM is

1. Right in the middle
2. Closer to the small ball
3. Closer to the big ball

$$X_{CAM} = \frac{X_1 m_1 + X_2 m_2}{m_1 + m_2} =$$

$$= \frac{0 \cdot 250 + 80 \cdot 600}{250 + 600} = \frac{80 \cdot 600}{850} = 56.5 \text{ cm}$$

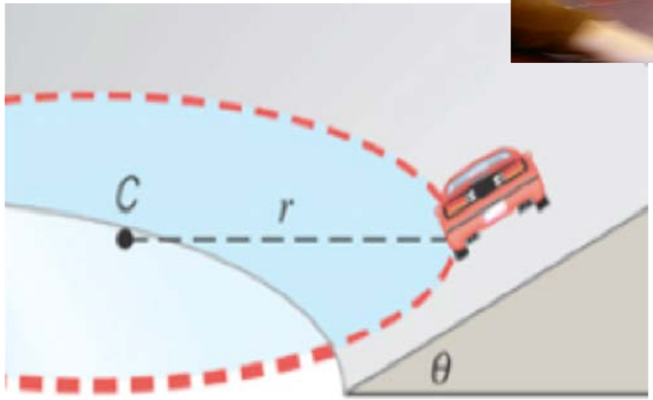
Circular Motion

=> **Trajectory is a circle**

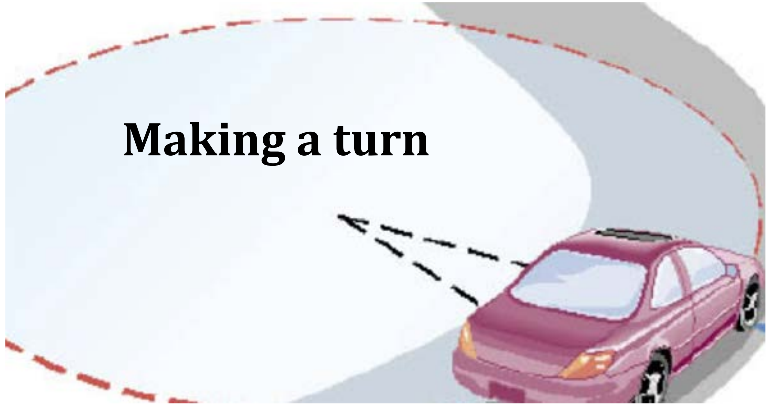

Devil's wheel

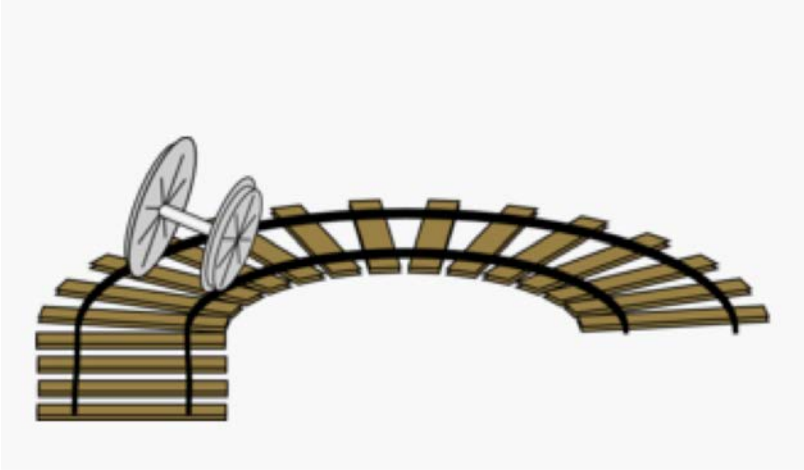
Roller coaster


Carnival ride



Banked tracks


Making a turn



Circular Motion

Type A:

***Uniform* (UCM)**

$|v|$ = const

Type B:

$|v| \neq$ const

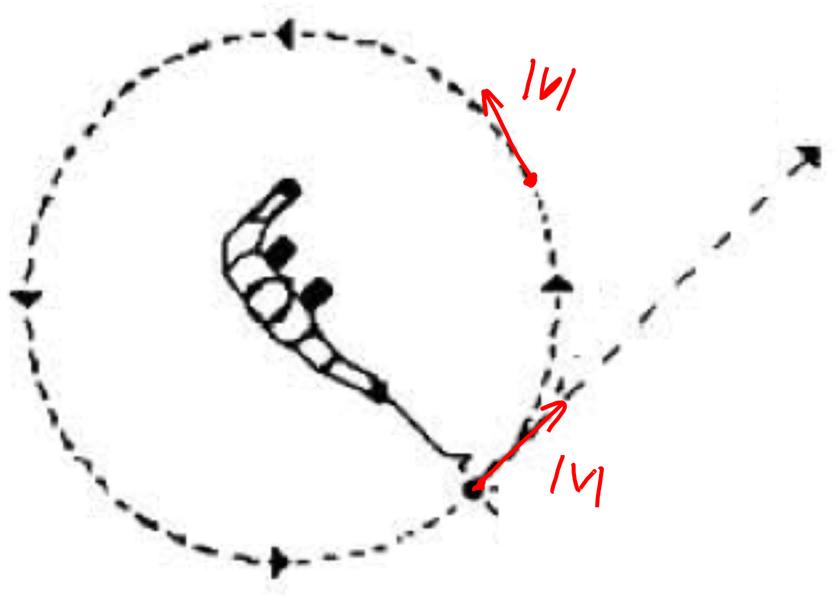
Not for us (yet)!

Uniform Circular Motion

Type A:
Uniform

$|v| = \text{const}$

<u>*Speed*</u> remains constant



Webassign: L12 Q3

When *speed* *is constant and NOT zero,* the motion is definitely …

1. Rhomboidal
2. Parabolic
3. Circular
4. Linear
5. 1^{st} or 2^{nd}
6. 2^{st} or 3^{d}
7. Not enough information
8. All of the above
9. None of the above

Webassign: L12 Q4

Webassign: L12 Q4

When *velocity* ***is constant and NOT zero,*** the motion is definitely …

1. Rhomboidal
2. Parabolic
3. Circular
4. Linear
5. 1^{st} or 2^{nd}
6. 2^{st} or 3^{d}
7. Not enough information
8. All of the above
9. None of the above

Webassign: L12 Q3 and Q4

Webassign: L12 Q3 and Q4

speed

When ***velocity*** ***is constant and NOT zero,*** the motion is definitely …

1. Rhomboidal
2. Parabolic
3. Circular
4. Linear
5. 1st or 2nd
6. 2st or 3d
7. Not enough information
8. All of the above
9. None of the above

v = const => $a = 0$ => $F_{net} = 0$ => N1L

When velocity *is constant and NOT zero,* the motion is definitely …

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2. Parabolic
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4. Linear
5. 1st or 2nd
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When *speed* ***is constant and NOT zero,*** the motion is definitely …

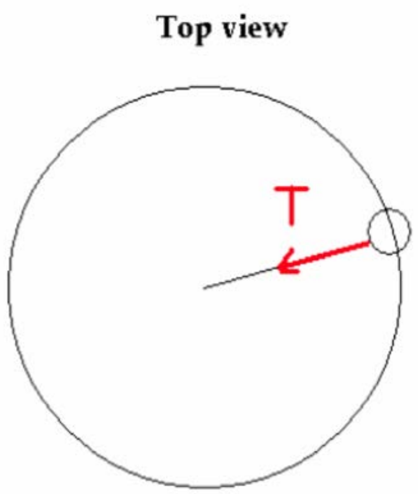
5. 1^{st} or 2^{nd}
6. 2^{st} or 3^{d}
7. **Not enough information**
8. All of the above

ove

Velocity ≠ Speed !!

Uniform Circular Motion (UCM)

$|v|$ = speed = const



You spin a small object attached to a string, making it move in circles with constant ***speed*** on a horizontal tabletop.

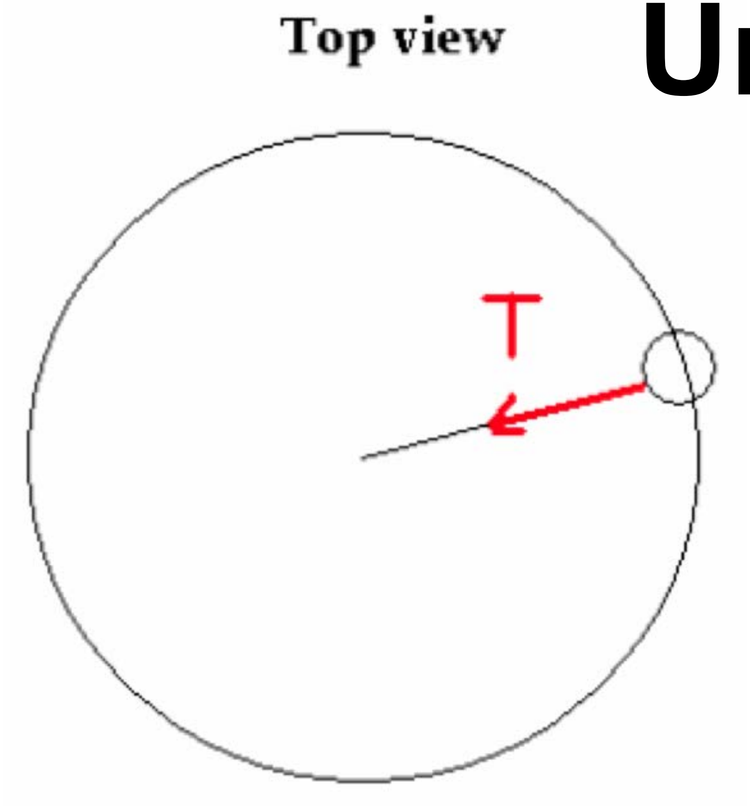
Webassign: L12 Q5

The acceleration of the object is **definitely**:

1. ZERO
2. NOT zero
3. Red

Uniform Circular Motion

$|v|$ = const

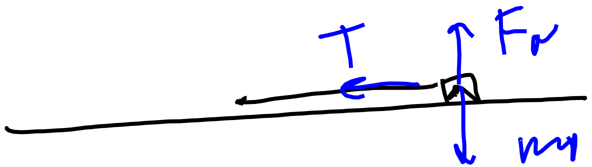


The acceleration of the object is definitely:

1. **ZERO**
2. **NOT zero**
3. **Red**

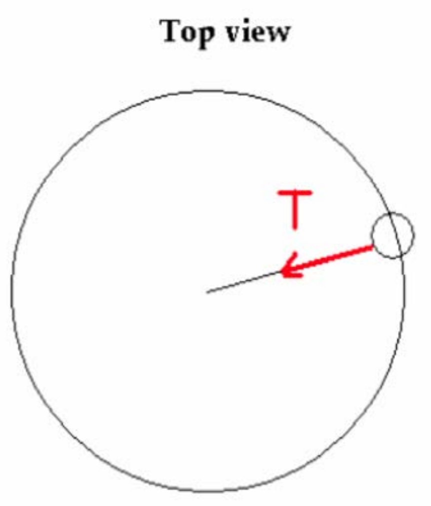
is $F_{net} = 0$?

NO



Uniform Circular Motion

$|v| = const$



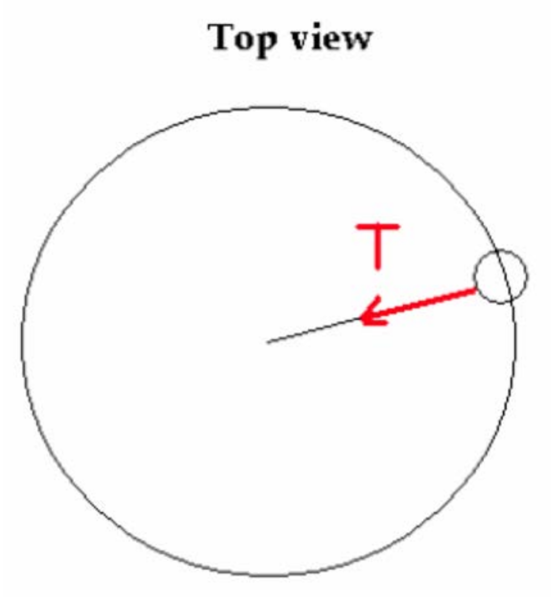
You spin a small object attached to string making it move in circles with constant speed on a horizontal tabletop.

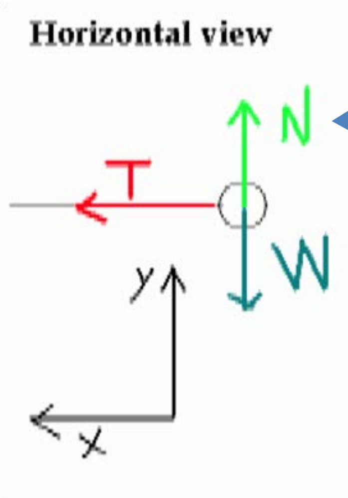
The acceleration of the ball is definitely :

1. ~~ZERO~~
2. NOT zero

Uniform Circular Motion

$|v|$ = const





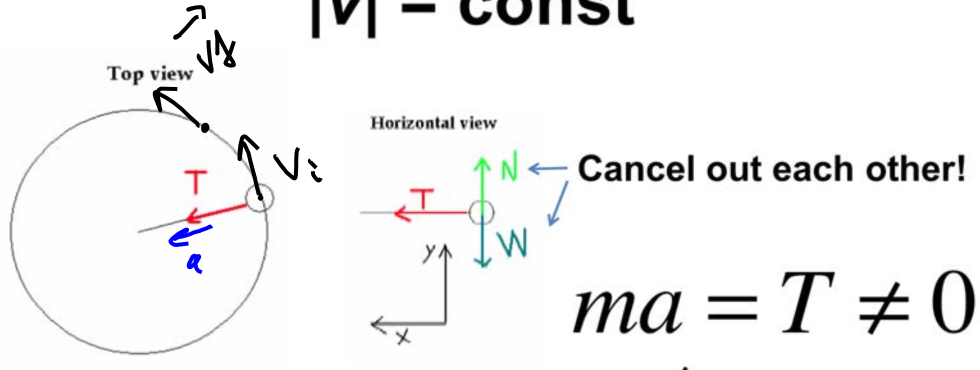
Cancel out each other!

$$ma = T \neq 0$$

$$\vec{v} \neq \text{const}$$

Uniform Circular Motion

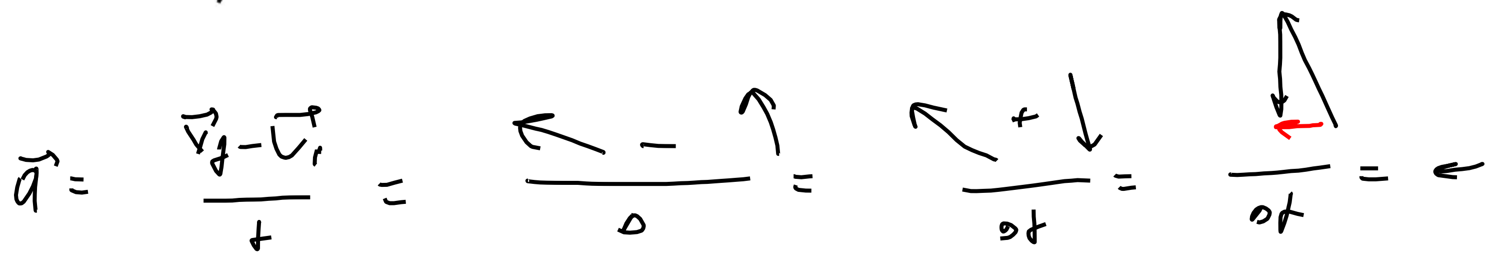
$|v| = \text{const}$



$$ma = T \neq 0$$

$\vec{v} \neq \text{const}$

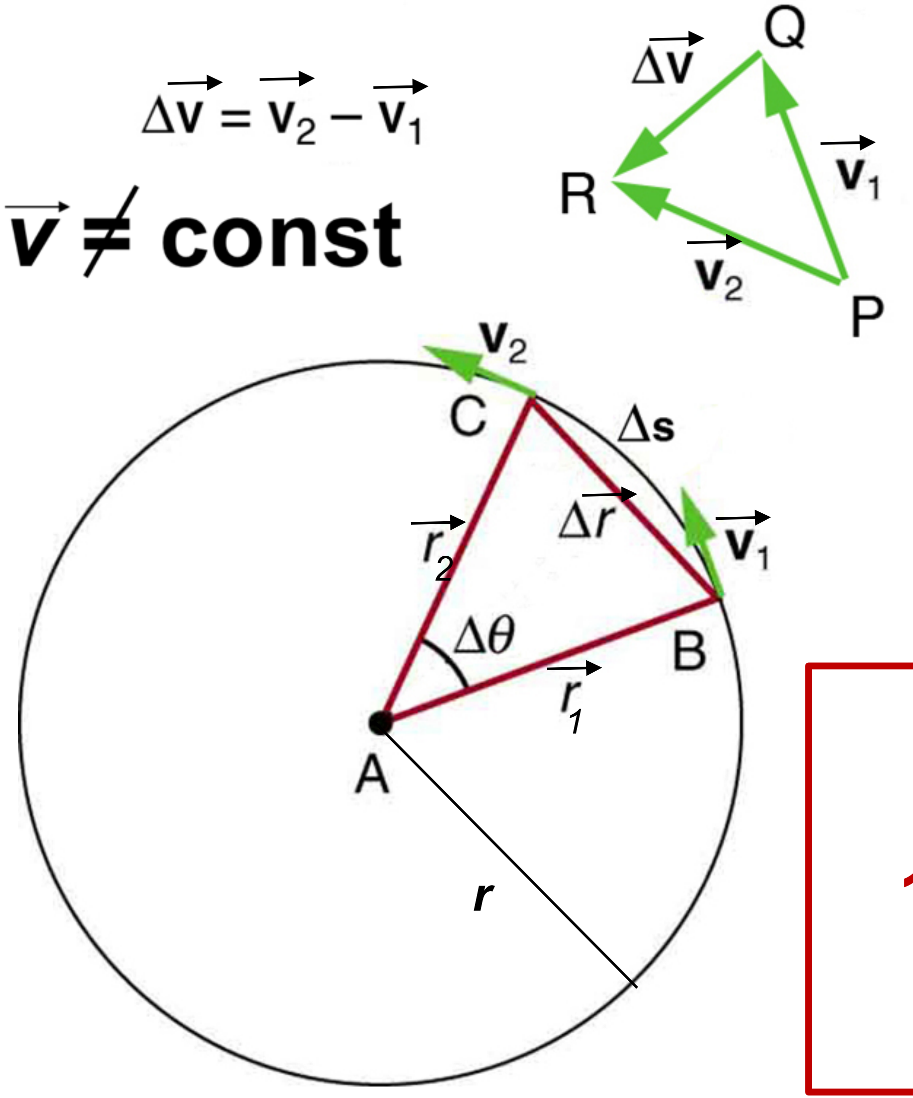
a **??** **t** **v**



Uniform Circular Motion (UCM)

$\vec{\Delta v} = \vec{v}_2 - \vec{v}_1$

$\vec{v} \neq$ **const**



Even if the *speed* is constant, the *velocity* changes!

That is why acceleration is NOT 0!

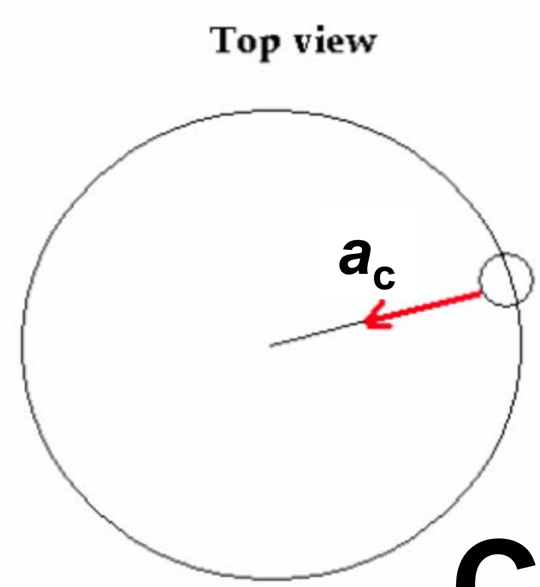
1. $|a_c| = \dfrac{v^2}{r}$

2. ALWAYS Points to the center!

Things to remember!

Uniform Circular Motion

|*v*| = const



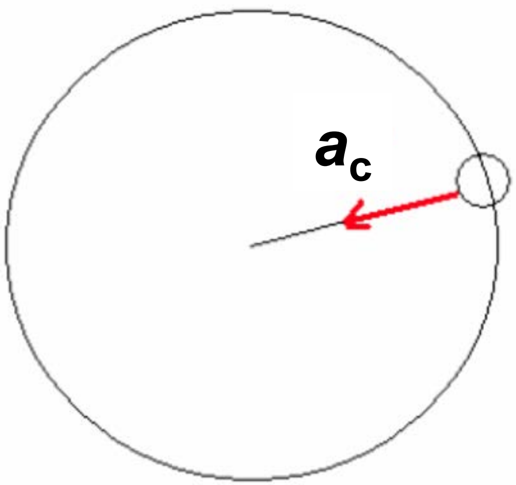
Points to the center!

(1) $\vec{F}_{NET} = m\vec{a}$

(2) $a_c = \dfrac{v^2}{r}$

Centripetal acceleration

(how fast velocity changes its direction)



ALL **problems related to a** ***uniform*** **circular motion of an object are solved by a combination of two equations:**

$$\vec{F}_{NET} = m\vec{a}$$

$$a_c = \frac{v^2}{r}$$

NB

The centripetal acceleration is the special form the acceleration has when an object is experiencing uniform circular motion. It is:

$$a_c = \frac{v^2}{r}$$

v = ANY !!

and is directed toward the center of the circle.

Newton's second law can then be written as:

$$\Sigma \mathbf{F} = ma = \frac{m\,v^2}{r}$$

|v|= const !!

(or at special locations)

Just a name for

A magical force "centripetal force" *do not exists!!!*

Inertia is the reason for a circular motion!

NB

A magical force "centripetal force" *do not exists!!!*

Inertia is the reason for a circular motion!

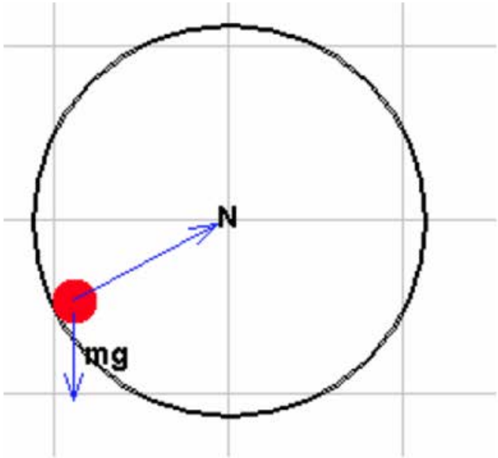
However, as just a *term*

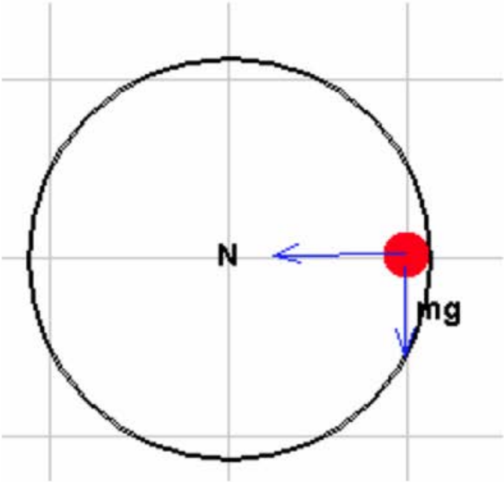
"centripetal force"

means ma_c

$$ma_c = \frac{m v^2}{r}$$

Vertical circular motion

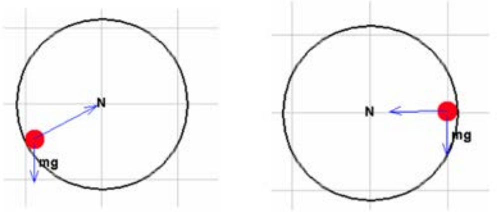




The situation of *vertical* circular motion is fairly common. Examples include:

- roller coasters ($a_c \leftrightarrow F_N$)
- water buckets ($a_c \leftrightarrow F_N$)
- cars traveling on hilly roads ($a_c \leftrightarrow F_N$)
- a ball on a string ($a_c \leftrightarrow F_T$)

Vertical circular motion



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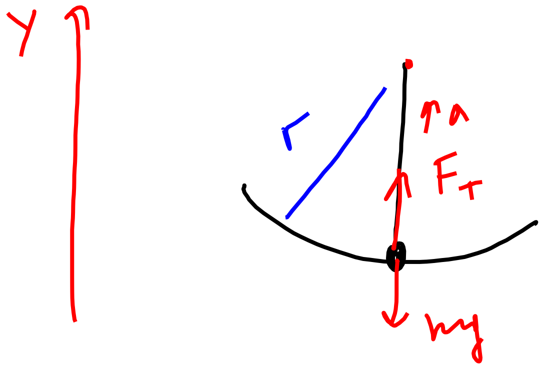


2 special cases

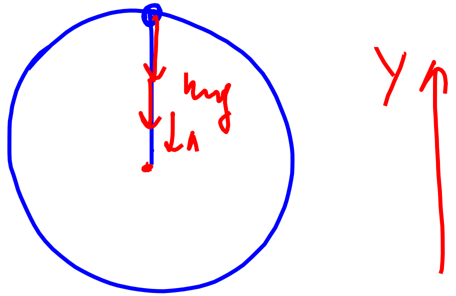
$$\vec{F}_{net} = m \cdot \vec{a}$$

$$F_T - mg = m \cdot a$$

$$a = a_c = \frac{v^2}{r}$$

$$F_T = mg + m\frac{v^2}{R}$$

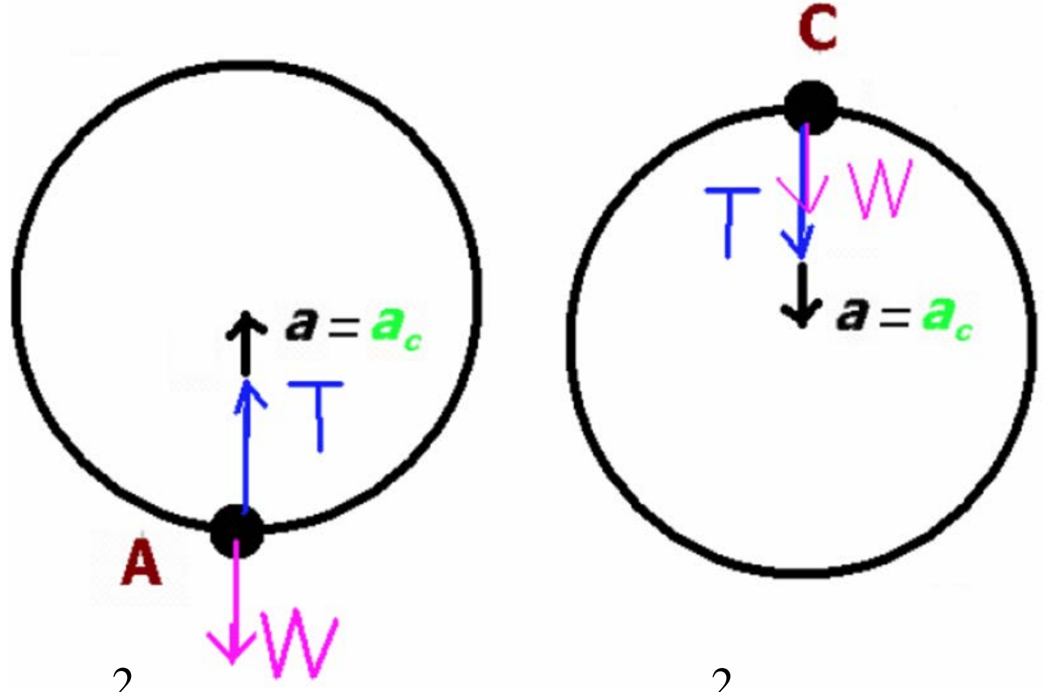
$a = a_c$



$$-F_T - mg = m(-a) \quad \times -1$$

$$F_T + mg = m\frac{v^2}{R}$$

$$F_T = m\frac{v^2}{R} - mg$$

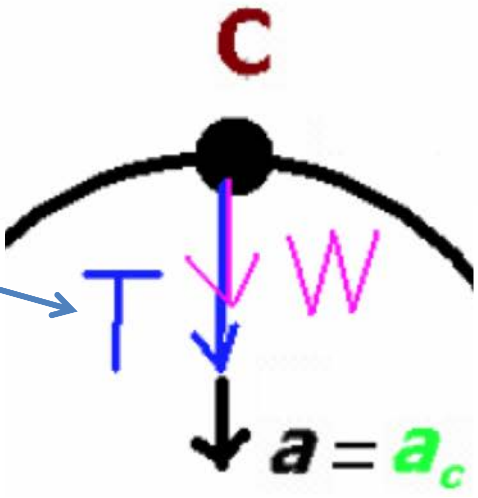


$$m\frac{v^2}{r} = T_A - mg \qquad -m\frac{v^2}{r} = -T_C - mg$$

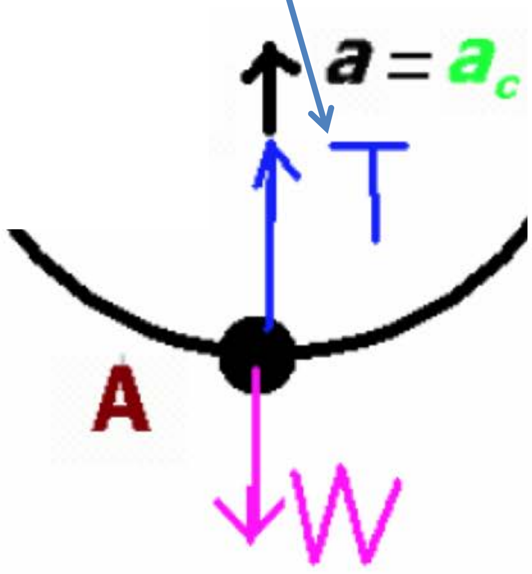
$$T_A = m\frac{v^2}{r} + mg \qquad T_C = m\frac{v^2}{r} - mg$$

T **could represent the tension in the string (if a ball is attached to it), or the normal force from a track (acting on a car), or any other force directed toward the center of a circular trajectory.**

The NATURE of this force does NOT matter!!



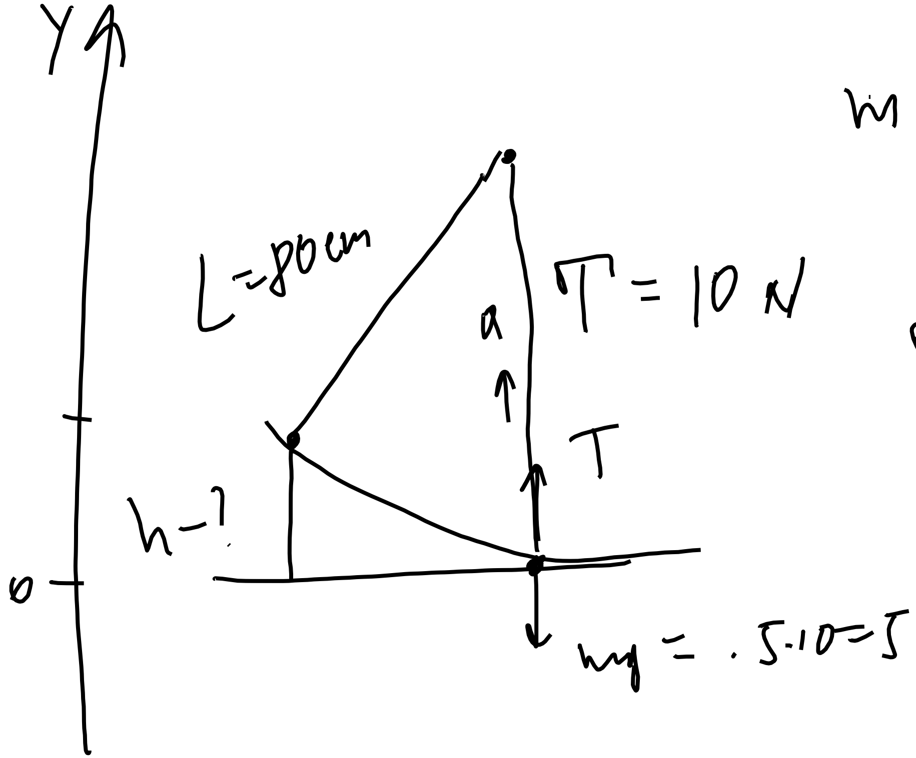
$$T_C = m\frac{v^2}{r} - mg$$



$$T_A = m\frac{v^2}{r} + mg$$

T **could represent the tension in the string (if a ball is attached to it), or the normal force from a track (acting on a car), or any other force directed toward the center of a circular trajectory.**

Calculate the **maximum** height.



$m = .5\ kg$

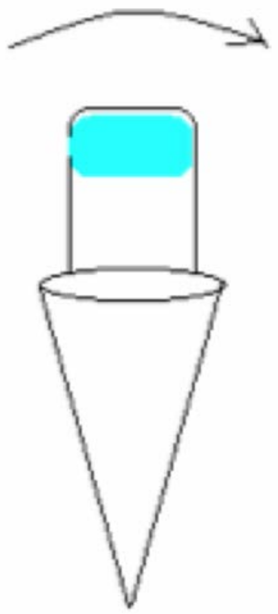
$T = mg + m\frac{V_c^2}{R}$

$10 = 5 + .5 \cdot \frac{V_c^2}{.8} \Rightarrow (10-5)\frac{.8}{.5} = V_c^2$

$V_c^2 = 8$

$KE_i + P_i = KE_f + P_f \Rightarrow \phi + mgh = \frac{mv^2}{2} + \phi$; $h = \frac{v^2}{2 \cdot g} = \frac{8}{2 \cdot 10} = .4$

A bucket with water is tied to a rope and whirled in a vertical circle with the radius of 0.5 m. What is the *minimum* speed of the bucket required to keep the water inside?



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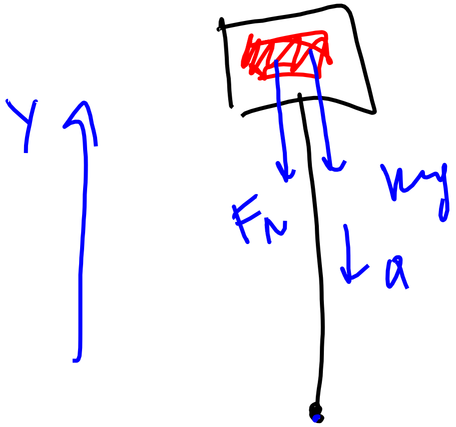


$F_N + mg = ma$

$V = V_c \Rightarrow F_N = \emptyset$

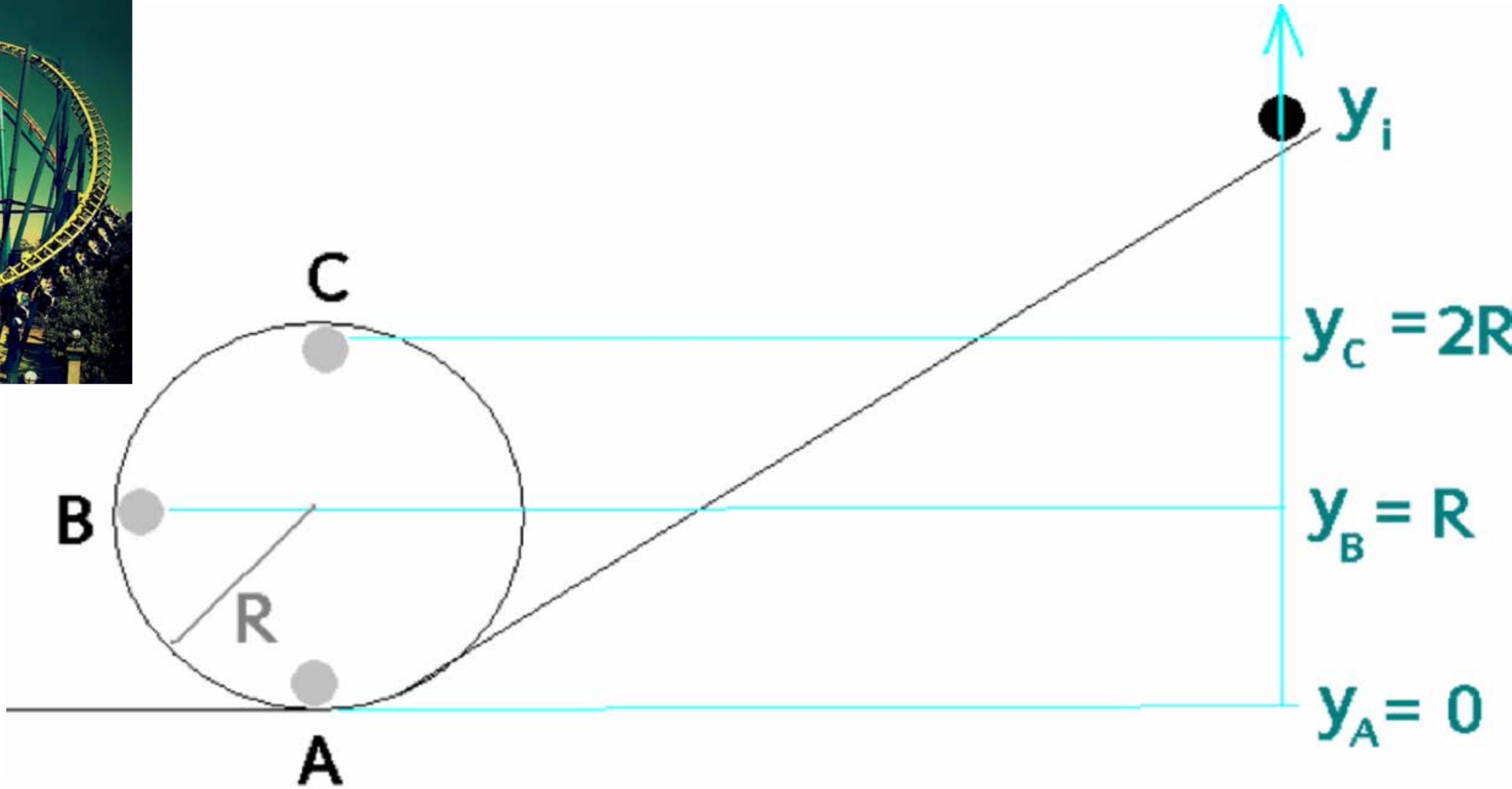
When $F_N = \emptyset$

$mg = ma$

$g = a \Rightarrow g = \frac{V_c^2}{R}$

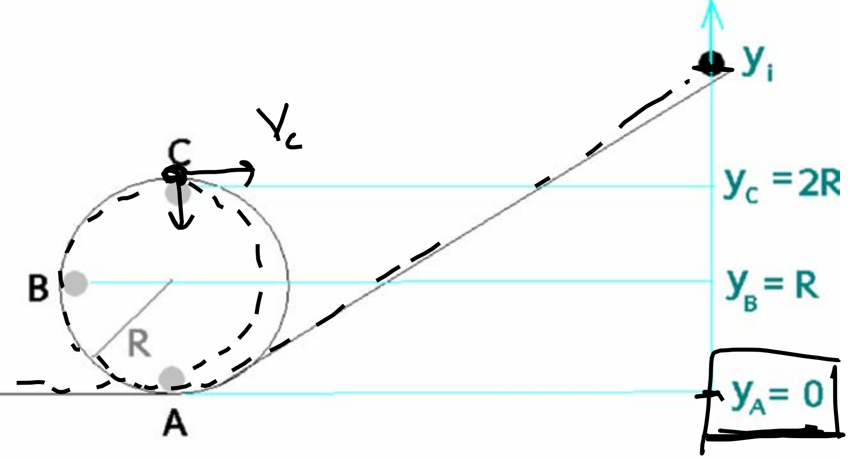
$V_c = \sqrt{gR} = \sqrt{10 \cdot .5} = \sqrt{5}$

$V_c = 2.2$ m/s



The ball is *sliding* down in the loop-the-loop demo.

(We are assuming there is no friction, so, the ball is *not* rolling; the rolling ball will be a *different* problem)



$Y_i = 0$

$H_c = Y_{i,c} \Rightarrow$ At C; $F_N = 0$

$\Downarrow$

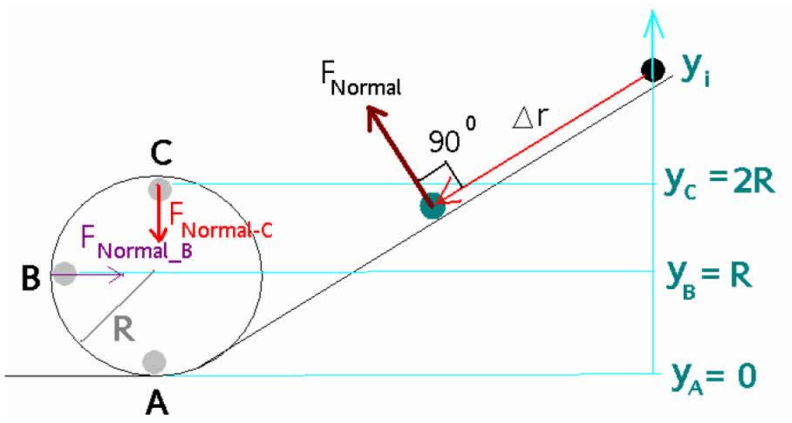
$mg = ma$

$V_c = \sqrt{R \cdot g}$

$\phi + mgY_i = \frac{m \cdot V_c^2}{2} + m \cdot g\, Y_c$

$Y_i = \frac{V_c^2}{2g} + Y_c = \frac{(\sqrt{Rg})^2}{2g} + Y_c = \frac{R}{2} + 2 \cdot R$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg(y_1 - y_2)$$



Point C (from y_i):

$y_C = 2R \quad v_i = 0$

$$\frac{1}{2}mv_C^2 = mg(y_i - 2R)$$

From the N II L:

$F_{Normal\text{-}C} + mg = ma_{c\text{-}C}$ and $a_{c\text{-}C} = v_C^2/R$

To get through the loop-the-loop, the ball has to get through the pint C, hence the minimum height y_i can be found from the condition $F_{Normal\text{-}C} = 0$ (the minimum possible values of the normal force). This gives us the known result, $a_{c\text{-}C} = g$.

That leads to the condition on the velocity $v_C^2/R = g$.

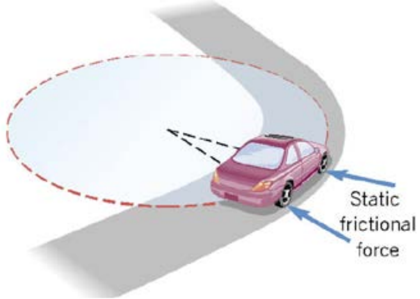
An finally, to the equation on the y_i $\frac{1}{2}mgR = mg(y_i - 2R)$

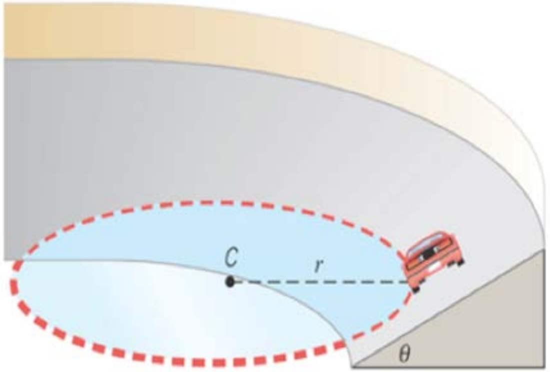
The solution is $y_i = 5R/2$ (for the sliding ball!)

Horizontal circular motion

Unbanked Curves

On an unbanked curve, the static frictional force provides the centripetal force.





A conic pendulum

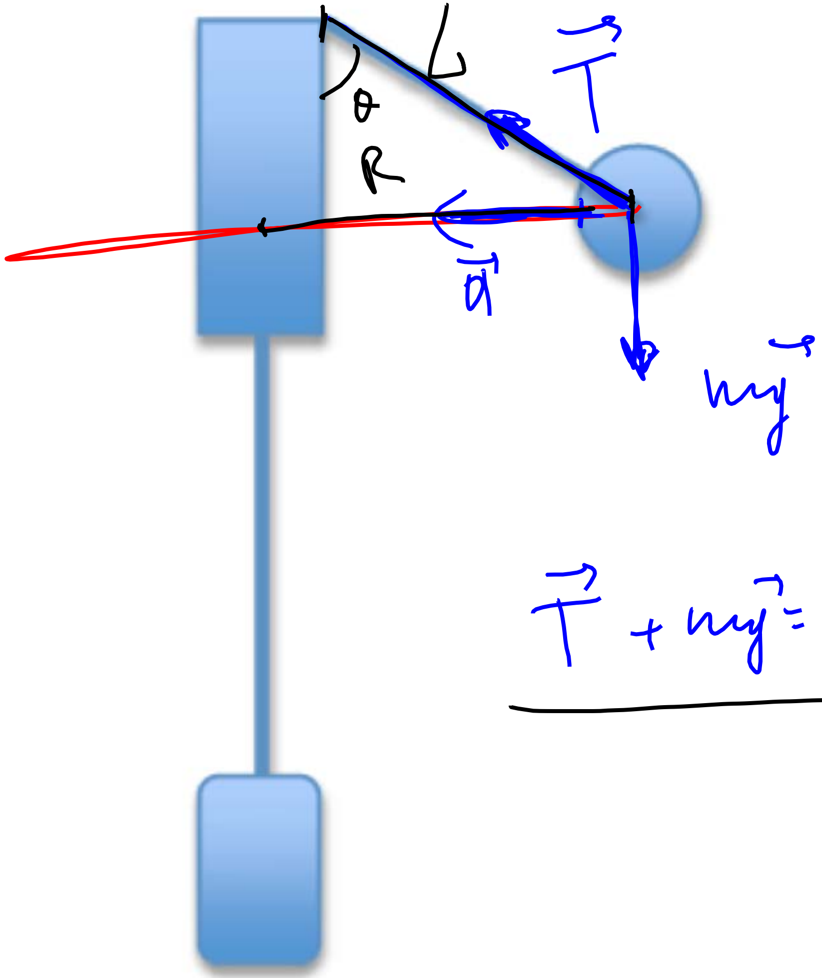
[Webassign: L12 Q6]

The <u>centripetal acceleration</u> of the ball *at the instant shown in the picture* points …

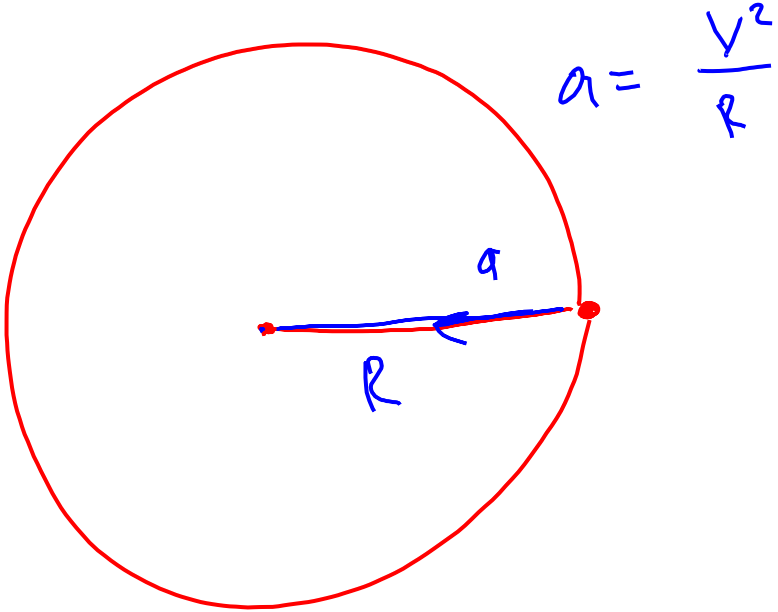
1. ↑
2. ↓
3. ←
4. →
5. ↘
6. ↖
7. ↗
8. ↙
9. None of the above
0. All of the above

A conic pendulum

A side view

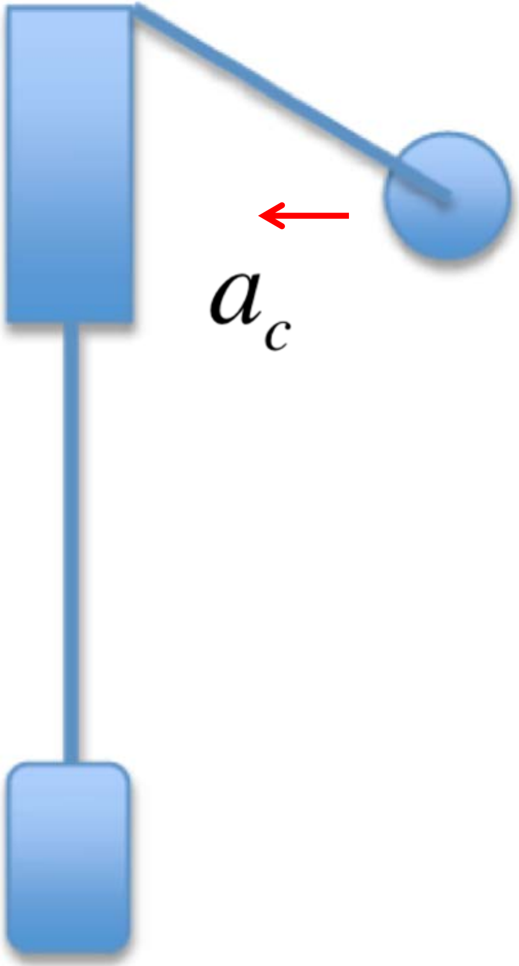


A top view



$$\vec{T} + m\vec{g} = m \cdot \vec{a}$$

A conic pendulum



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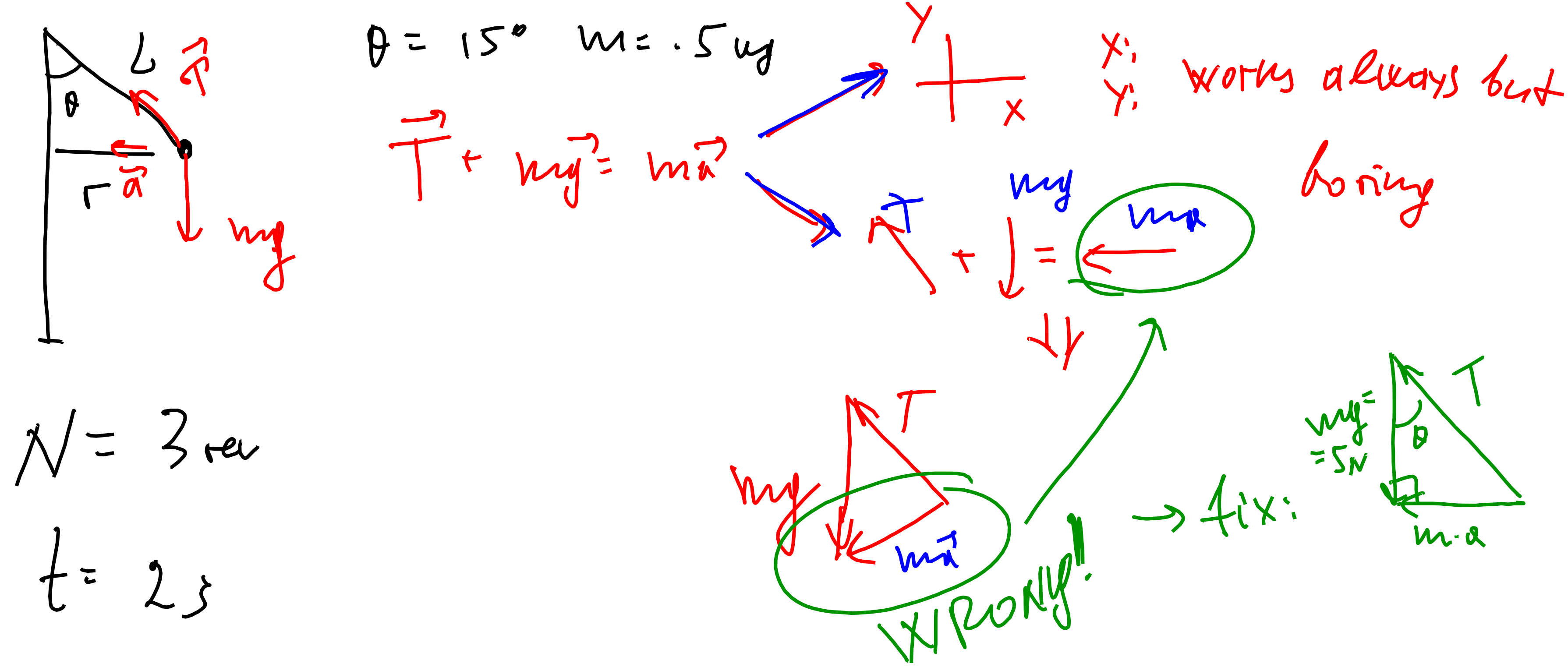
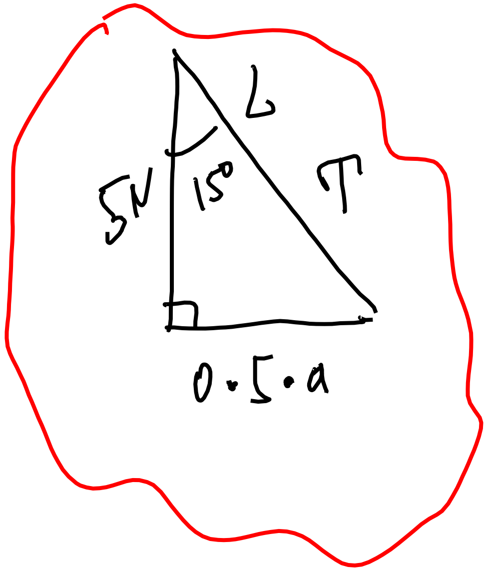
1. ↑
2. ↓
3. ←
4. →
5. ↘
6. ↖
7. ↗
8. ↙
9. None of the above
0. All of the above

The mass of the weight is 500 grams, the string make angle of 15^0 to the vertical. The weight makes 3 revolutions every 2 second. Find … everything.





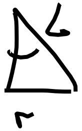
$$\Longrightarrow \quad \frac{5}{T} = \cos 15° \Rightarrow T = \frac{5}{\cos 15} = 5.2\,N$$

$$\frac{0.5 \cdot a}{5} = \tan 15° \Rightarrow a = 10 \cdot \tan 15° = 2.7\ m/s^2$$

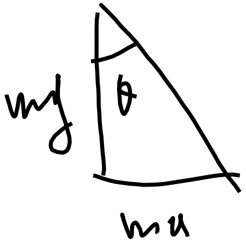
$$a = \frac{V^2}{r}; \qquad V = \frac{L}{t} = \frac{3 \cdot 2\pi r}{2} = 3\pi \cdot r$$

$$2.7 = a = \frac{(3\pi r)^2}{r} = \frac{9 \cdot \pi^2 \cdot r^2}{r} = 9 \cdot \pi^2 \cdot r$$

$$r = \frac{2.7}{9 \cdot \pi^2}$$



$$\frac{r}{L} = \sin\theta ;\ L = \frac{r}{\sin\theta} = \ldots$$

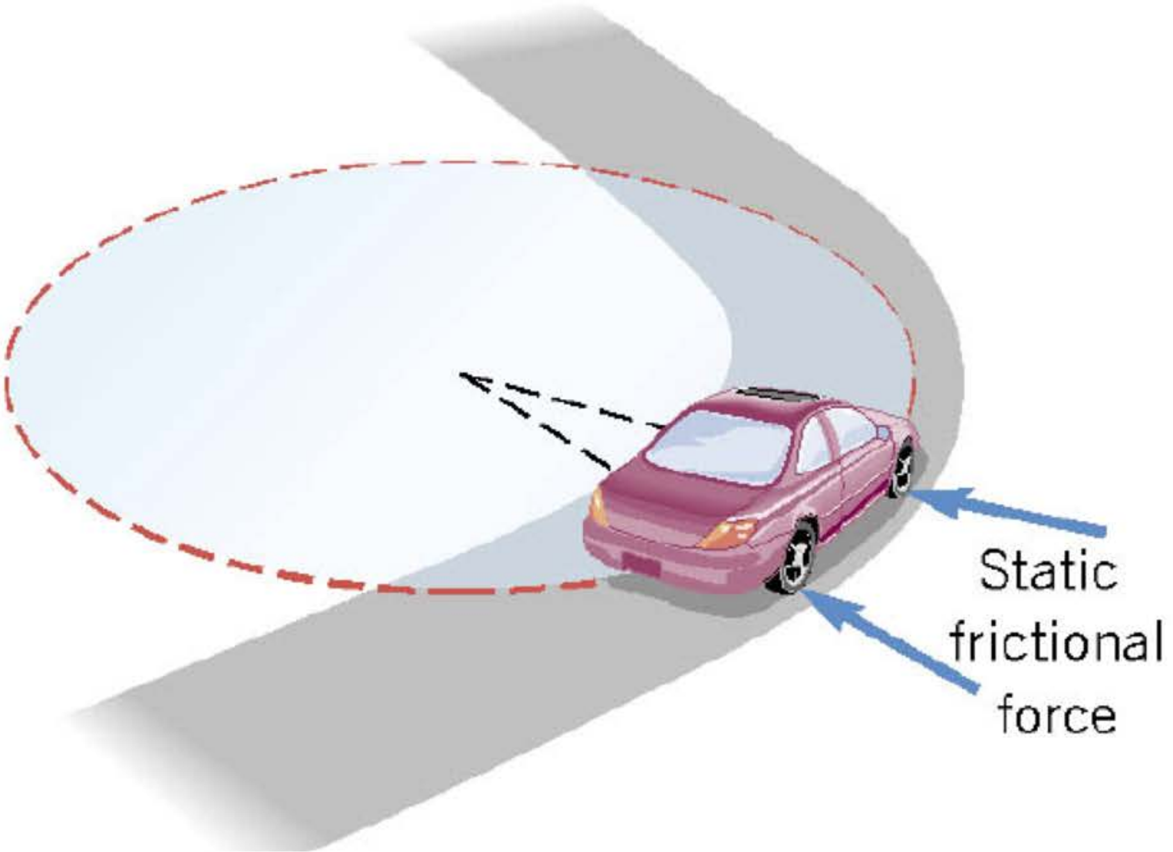


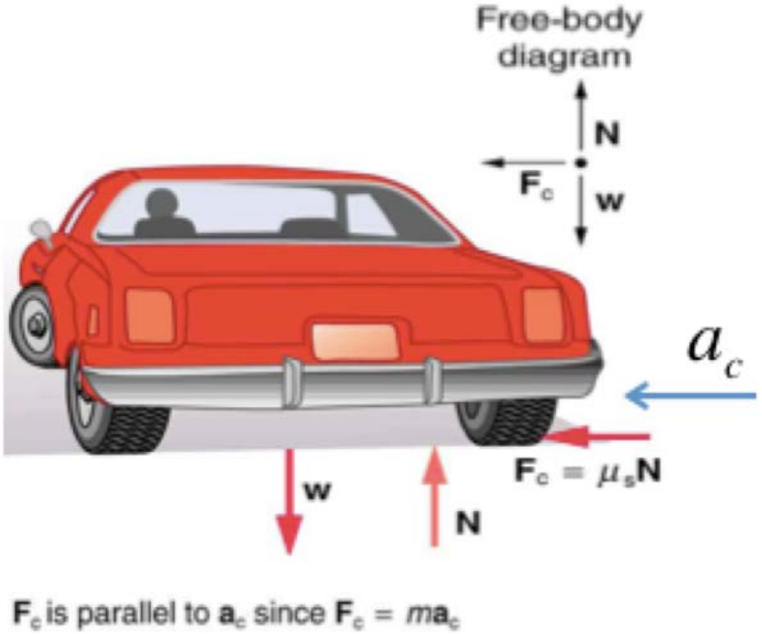
$$\frac{ma}{mg} = \tan\theta$$

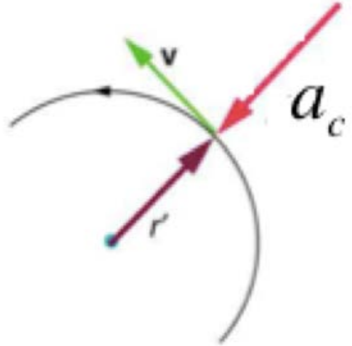
$$a = g \cdot \tan\theta$$

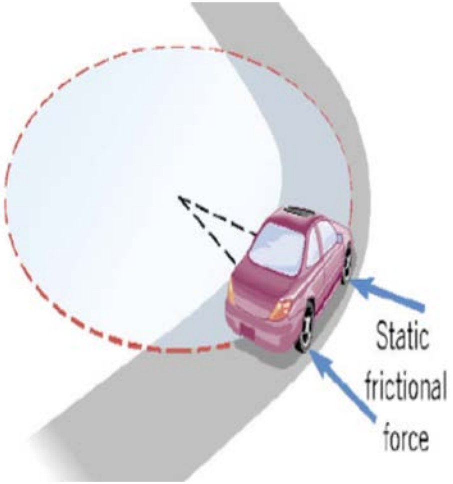
Unbanked Curves

On an unbanked curve, the static frictional force provides the centripetal force.



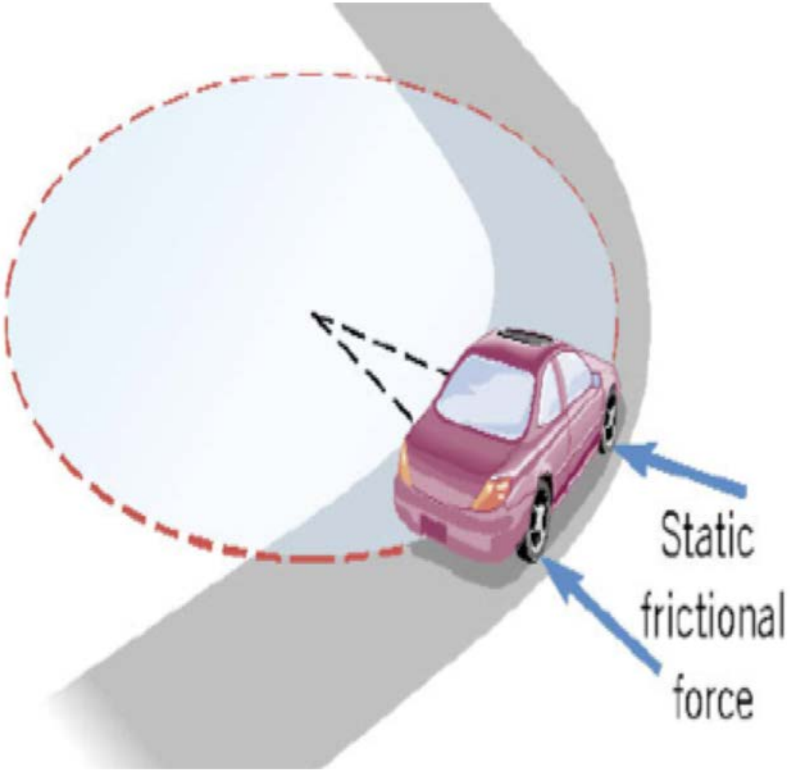




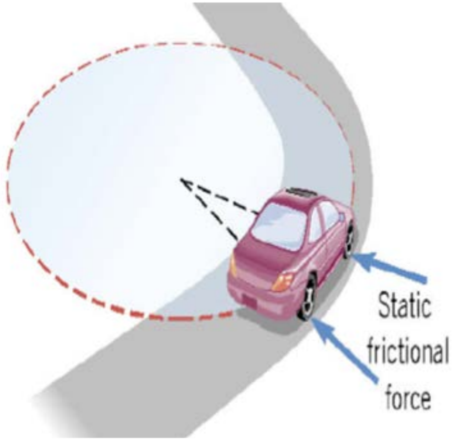


A car on a level ground is moving away and turning to the left. The "centripetal force" causing the car to turn in a circular path is due to friction between the tires and the road.

A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.



What is the maximum speed a car can have to negotiate a turn of radius of 100 m on a surface with the coefficient of static friction $\mu = 0.9$?



What is the maximum speed a car can have to negotiate a turn of radius of 100 m on a surface with the coefficient of static friction $\mu = 0.9$?

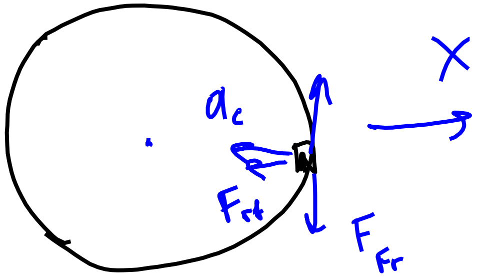
N2L: x: $F_{fr} = ma$

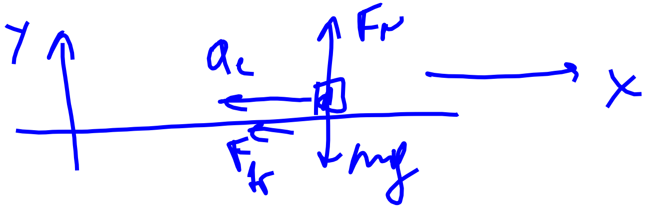
critical ⇓

$$F_{fr} = F_{fr\,max} = \mu \cdot F_N = m \cdot \frac{v_c^2}{R}$$

⇓

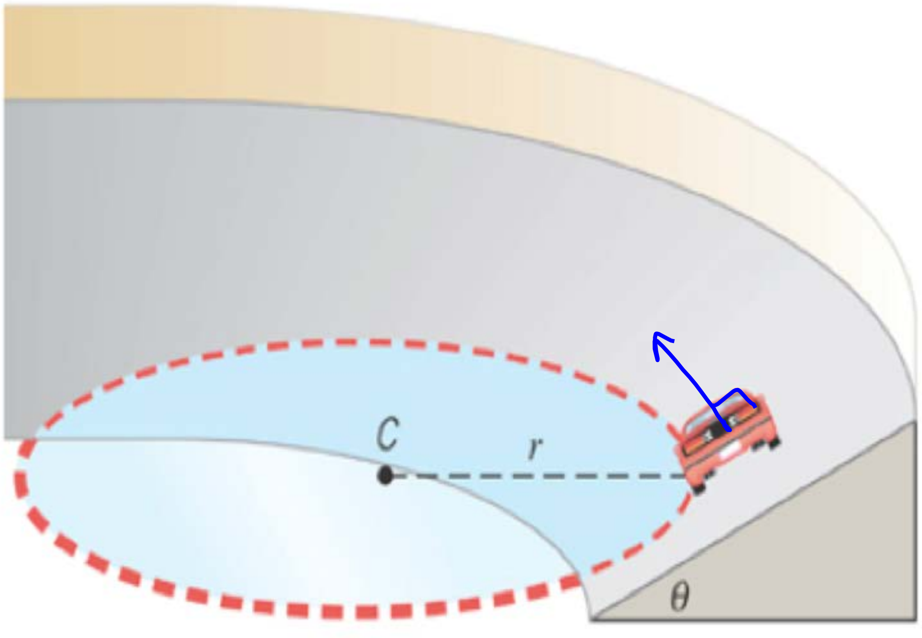
$$\mu \cdot mg = m \cdot \frac{v_c^2}{R}; \quad v_c = \sqrt{\mu \cdot R \cdot g} = \sqrt{.9 \cdot 100 \cdot 10} = 30 \text{ m/s}$$



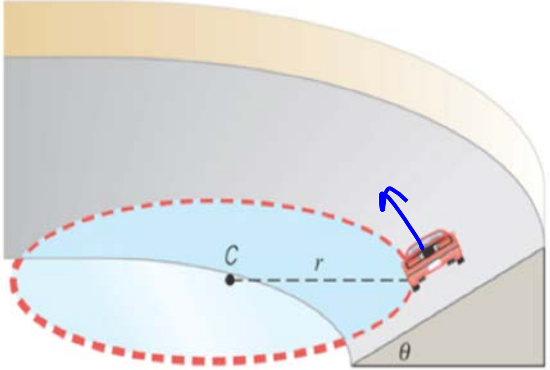


The turns at the Daytona International Speedway have a maximum radius of 316 m and are banked at 31^0. Suppose the

turns were frictionless. At what speed would the cars have to travel around them?



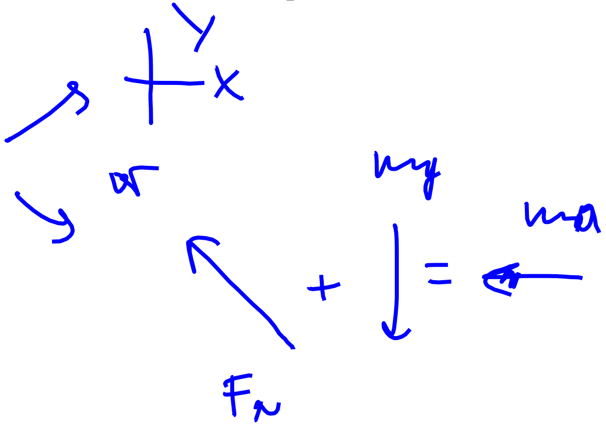
A car is driving on a banked curve at a constant speed. Calculate the *optimal* speed for the car if the radius of the circular trajectory is 316 m and the banked track is at 31^0 to the horizontal direction.

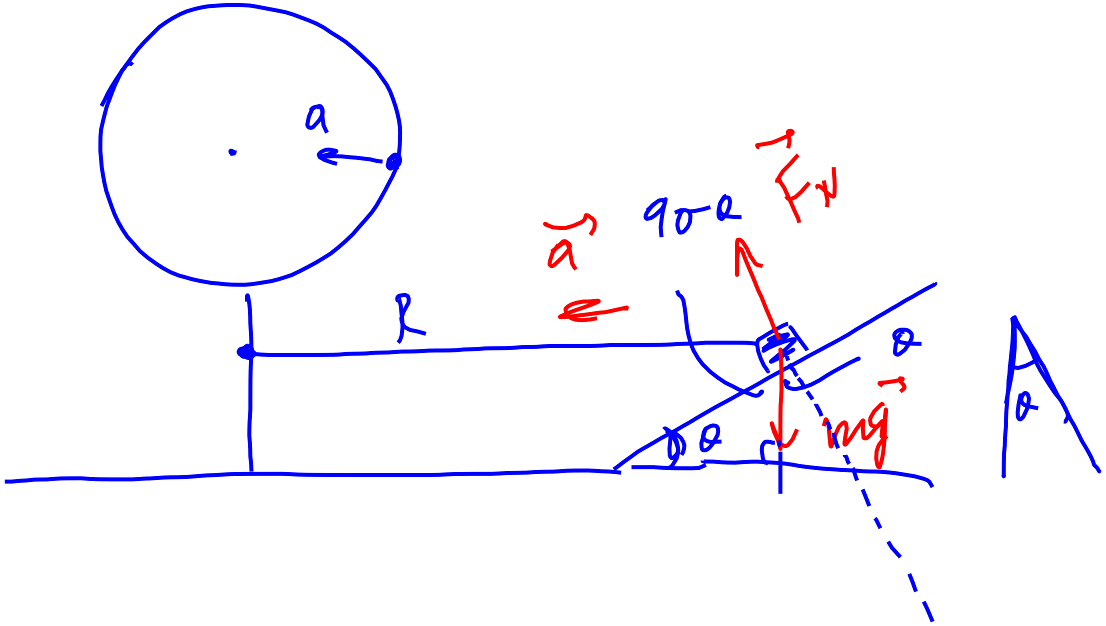


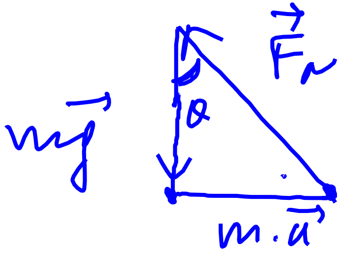
Draw **FBD** and write 2NL for a car driving on a banked curve ($|v|$ = const).

$\Theta = 31^0$, $R = 316$ m

N2L: $\vec{F}_N + m\vec{g} = m \cdot \vec{a}$





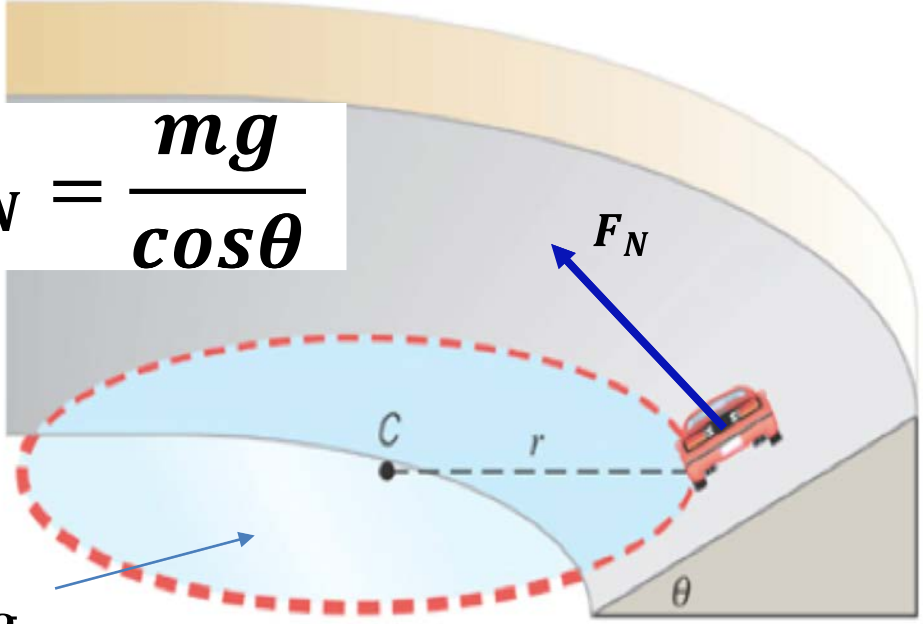


$\frac{ma}{mg} = \tan\theta$

$\frac{mg}{F_N} = \cos\theta$

$F_N = \frac{mg}{\cos\theta}$!

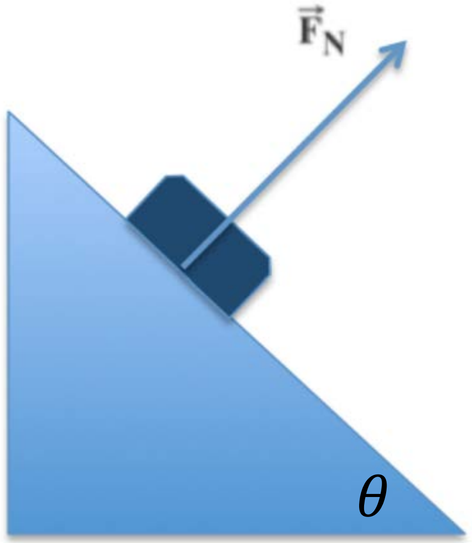
$$F_N = \frac{mg}{cos\theta}$$



$$\frac{mg}{F_N} = \cos\theta$$

is NOT!

$$F_N = mgcos\theta$$

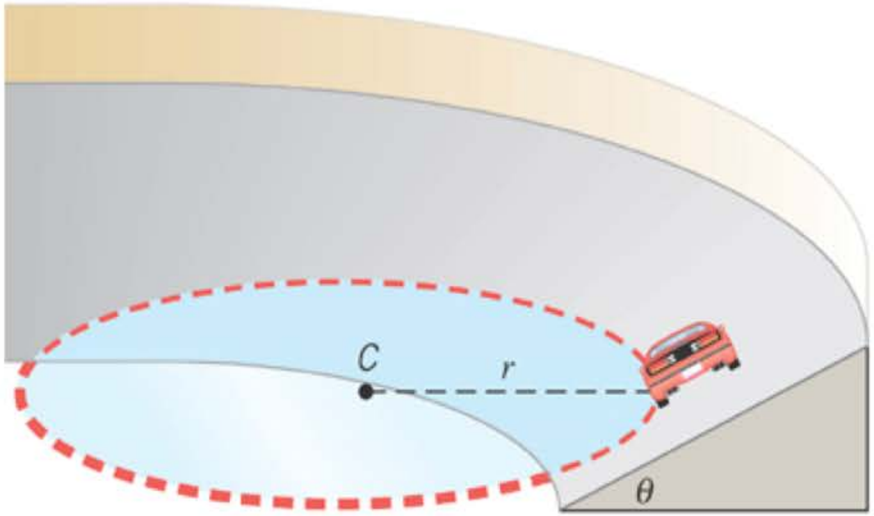


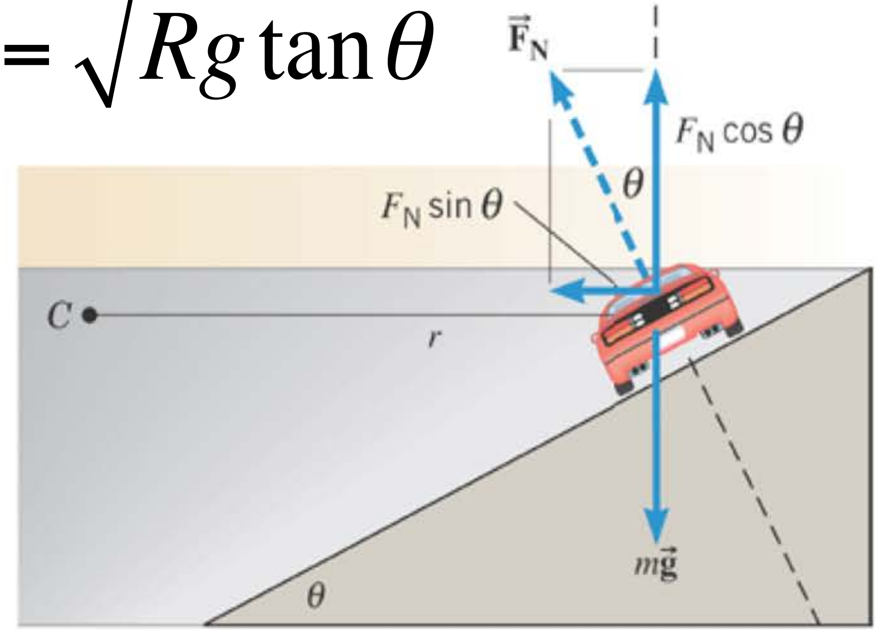
$$\frac{F_N}{mg} = \cos\theta$$

Draw FBD and write 2NL for a car driving on a banked curve (|*v*| = const).

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight

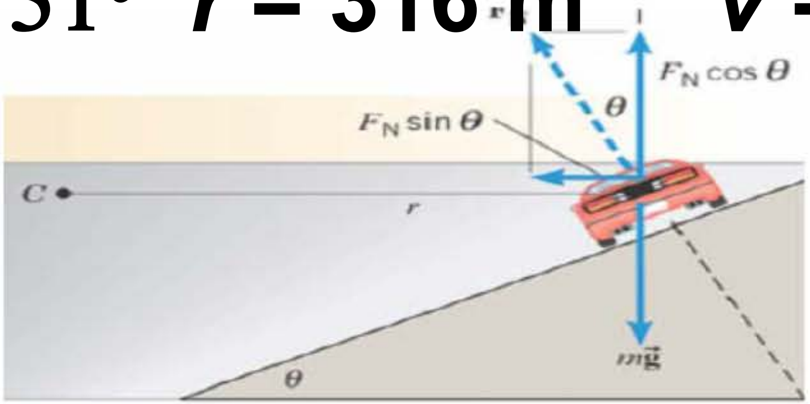
When the speed is *optimal* $v = \sqrt{Rg\tan\theta}$





No friction is needed for moving with no sliding up or down!

$\theta = 31^0$ **r = 316 m** **v = ?**



The turns at the Daytona International Speedway have a maximum radius of 316 m and are steely banked at 31°.

Suppose these turns were frictionless.

As what speed would the cars have to travel around them?

$$\frac{mg}{F_N} = \cos\theta \qquad ma_c = F_N \sin\theta$$

Let's write the Newton's II law in the projection on the direction of the centripetal acceleration: $ma_c = F_N \sin\theta$, hence $mV^2/R = F_N \sin\theta$

Now let's write the Newton's II law in the projection on the direction of the force of gravity: $m*0 = mg - F_N \cos\theta$

Now we can solve this system for the variable V.

$$V = \sqrt{\frac{RF_N \sin\theta}{m}} = \sqrt{\frac{gRF_N \sin\theta}{mg}} = \sqrt{\frac{gRF_N \sin\theta}{F_N \cos\theta}} = \sqrt{\frac{gR\sin\theta}{\cos\theta}} = \sqrt{Rg\tan\theta}$$ **= 43 m/s**