Lab 6 is in SCI 134

Please, login into webassing, locate LectureMCQ_L14 (PY105) and answer question 1 (**but ONLY Q1!**).

Pleas sign in using the sign-in sheets on the bench. Thank you

Note: exam room change:
Exams 2, 3 take place in STO B50

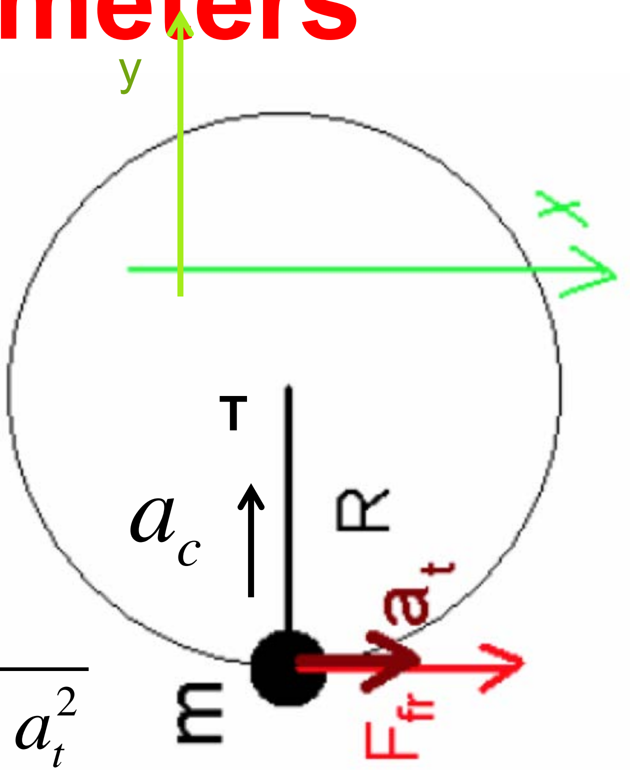
Good morning!

Circular motion with $|v| \neq$ const for a small object in terms of angular parameters

$$\omega = v / r$$

$$a_c = \frac{v^2}{R}$$

$$T = ma_c$$



$$a = \sqrt{a_c^2 + a_t^2}$$

$$F_{fr} = ma_t$$

$$\boxed{\frac{a_t}{r} = \alpha}$$

$$F_{fr} = m\alpha r$$

$$F_{fr} r = m\alpha r^2 = \boldsymbol{mr^2\alpha}$$

Because of …

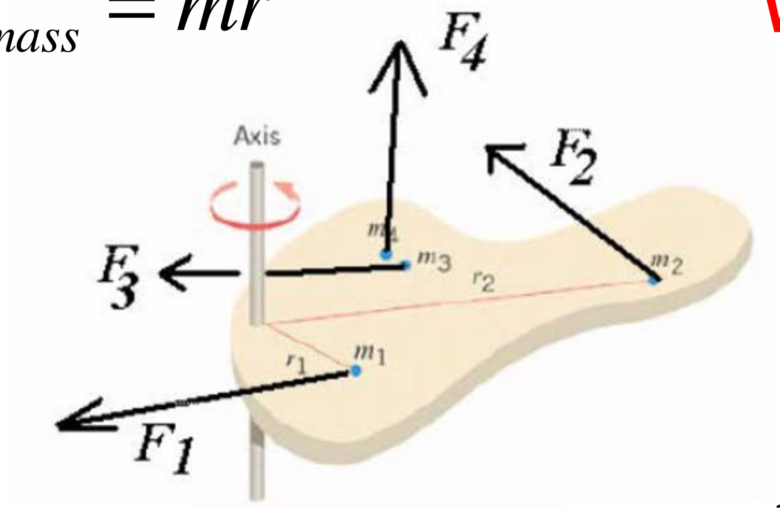
$\boldsymbol{\tau = I\alpha} \rightarrow$ **The same for ALL points!**

Newton's Second Law for Rotation

$$\tau_{one\ \mathrm{point}} = (mr^2)\alpha$$

$$I = \Sigma mr^2 = CMR^2$$

$$I_{small\,mass} = mr^2$$



When the axis ***passes*** CofM (!):

$$I_{rod} = \frac{1}{12} mL^2$$

$$I_{sph} = \frac{2}{5} mr^2$$

$$I_{hollow\ cyl} = I_{ring} = mr^2$$

$$\vec{\tau}_{Net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots$$

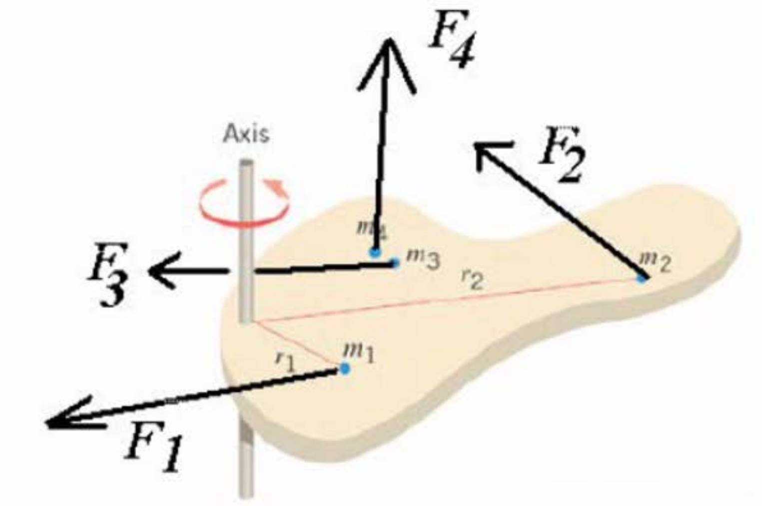
$$\boxed{\vec{\tau}_{Net} = I\vec{\alpha}}$$

$$I_{disc} = I_{cyl} = \frac{1}{2} mr^2$$

Newton's Second Law for Rotation

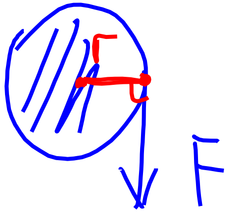
$$I_{hollow\ cyl} = I_{ring} = mr^2$$

$$I_{disc} = I_{cyl} = \frac{1}{2}mr^2$$



$|\tau| = r \cdot F_y$

$F_y = F$



$F \Rightarrow \tau = F \cdot r$

$\alpha = \frac{\tau}{I}$

$I \uparrow \quad \alpha \downarrow$

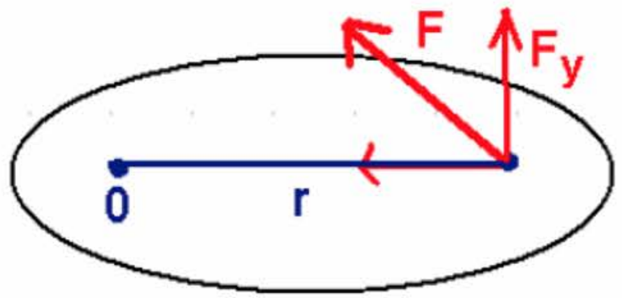
$\tau = \text{const}$

$a = \frac{F}{m}$

$$\vec{\tau}_{Net} = I\vec{\alpha}$$

! The *actual* (physical) axis of rotation is not always equal to the *abstract* (mathematical) axis of rotation. !

Torque - DEFINITION: *magnitude*



The torque τ of the force F is a vector with the magnitude of

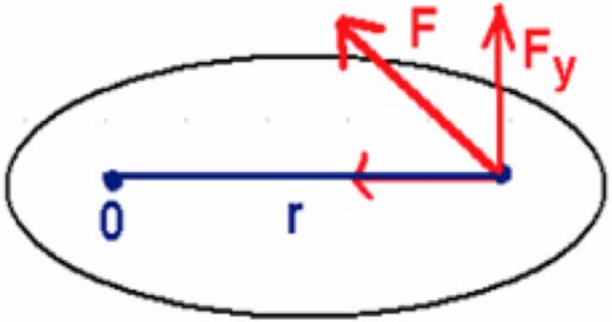
$$\boxed{|\tau| = r\,|F_y|}$$

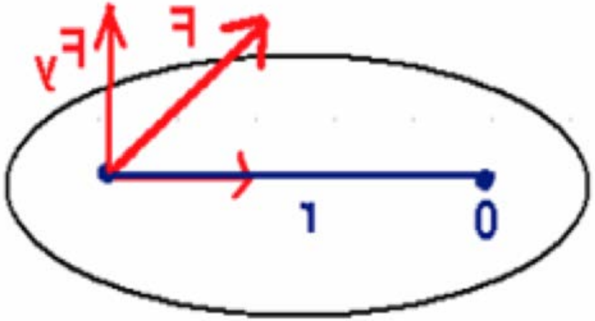
r is the distance from the point of application of the force to the axis of rotation

$\mathbf{F_y}$ is the component of the force perpendicular to the line connecting the point of application and the axis of rotation.

Torque - DEFINITION: *direction*

CCW => "+" **CW => "-"**

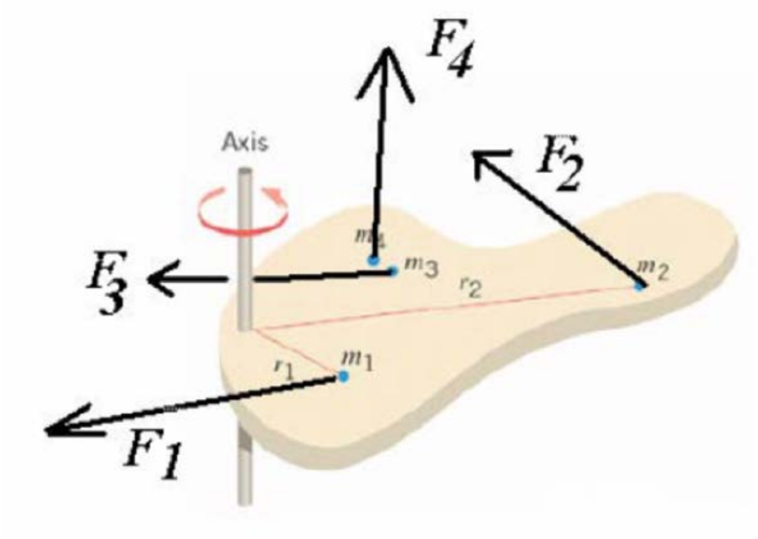




! **The *actual* (physical) axis of rotation is not always equal to the *abstract* (mathematical) axis of rotation.** !

When solving a problem – *select* the axis of rotation and do NOT change it until the *end*!

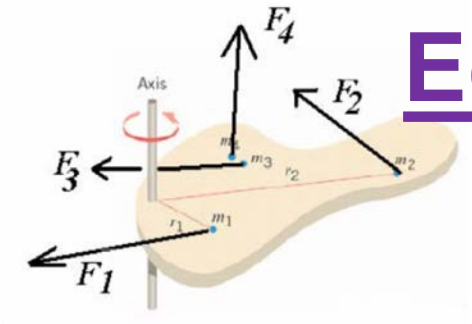
"<u>torque</u>" is a shortcut for



"THE TORQUE OF <u>this</u> FORCE REALTIVE TO <u>this</u> AXIS"

The actual (physical) axis of rotation is not always equal to the abstract (mathematical) axis of rotation.

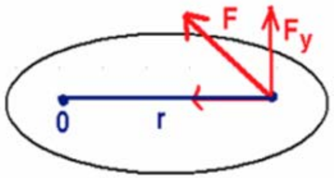
<u>You can choose the axis of ration!</u>



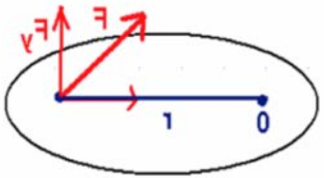
Equilibrium => A rigid object is at rest

$$|\tau| = r\,F_y$$

CCW => "+"



CW => "-"



$$\vec{\tau}_{Net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots$$

$$\vec{\tau}_{Net} = 0$$

and

$$\omega = 0$$

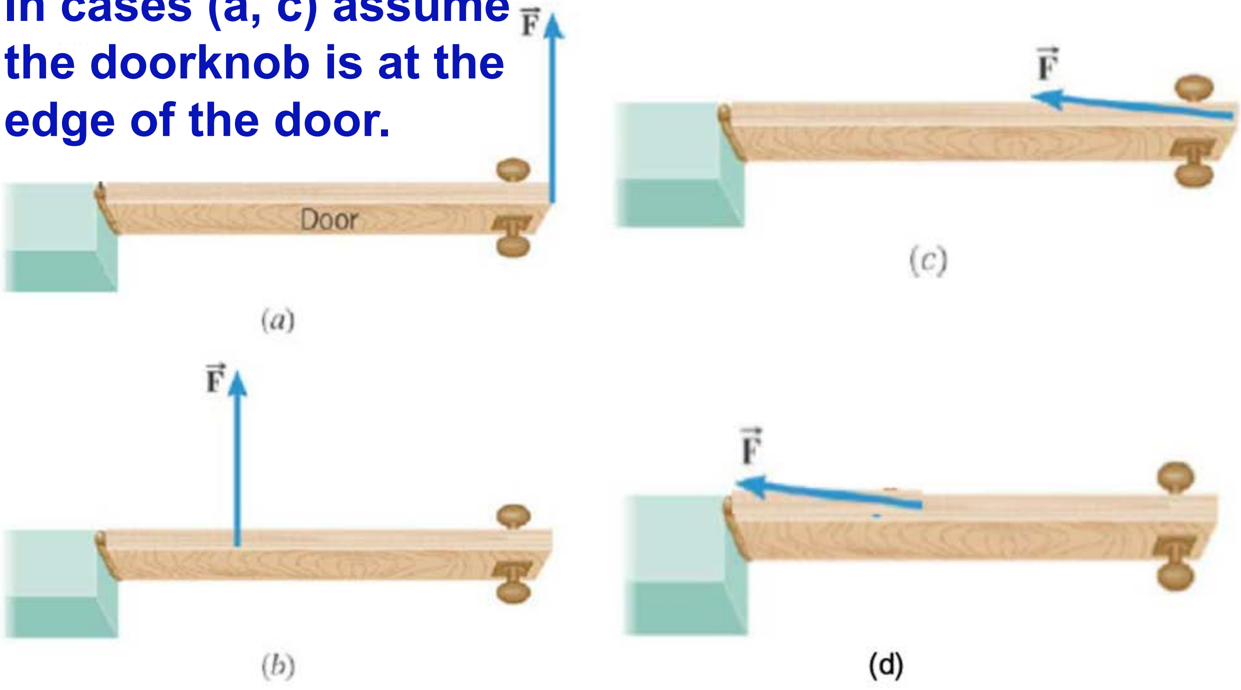
ALL problems on static equilibrium of a rigid object start from this equation, called torque-balance equation (TBE).

The actual (physical) axis of rotation is not always equal to the abstract (mathematical) axis of rotation.

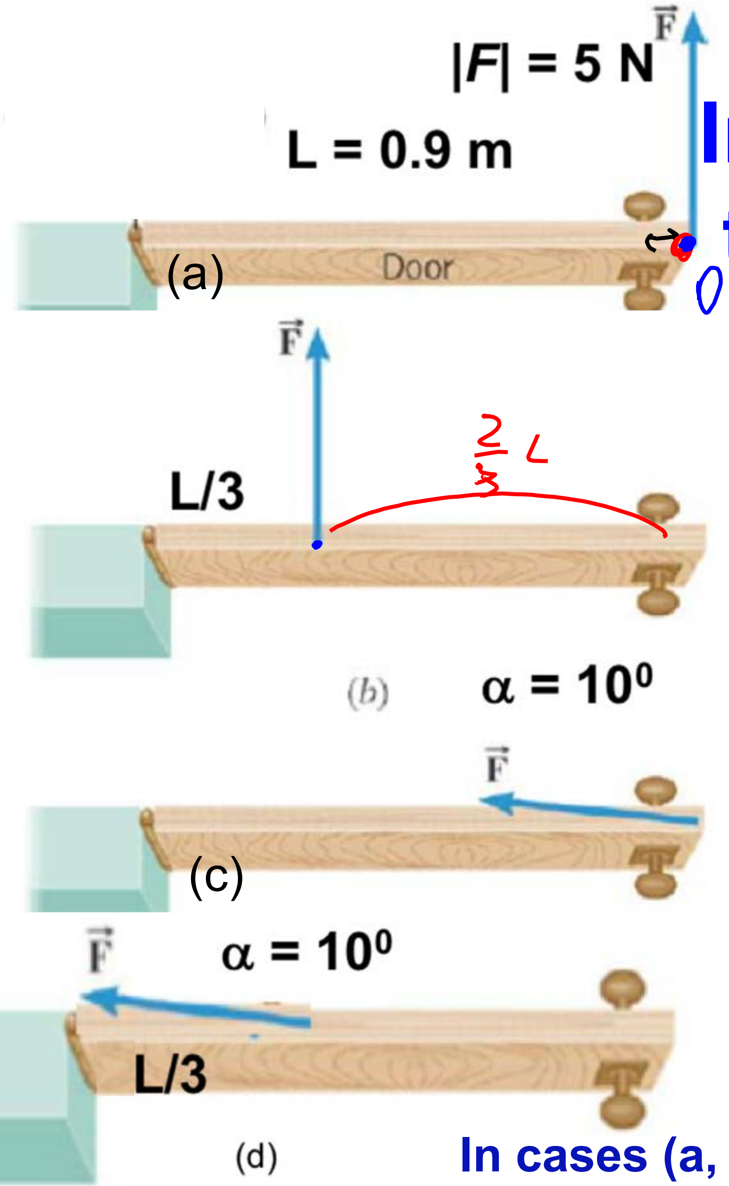
<u>You can choose and keep your axis!</u>

Calculate the torque relative to the *<u>doorknob</u>*.

In cases (a, c) assume the doorknob is at the edge of the door.



$|F|$ = 5 N, the door is 0.9 m long. In cases (b, d) the force is 2/3 closer to the hinge as in case (a); in cases (c, d) the angle is 10^0. Calculate the applied torque.

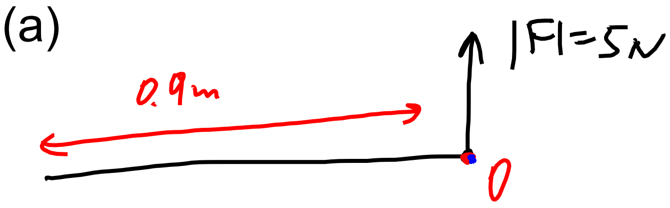


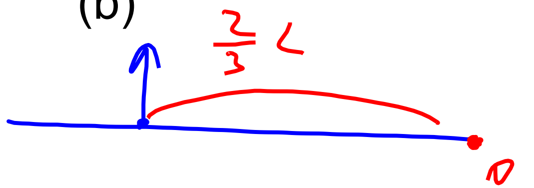
Webassign: L14 Q2

In case (d), relative to the *doorknob* torque is 1. > 0 2. = 0 3. < 0

CCW => "+" CW => "-"

$|\tau| = r \cdot F_y$





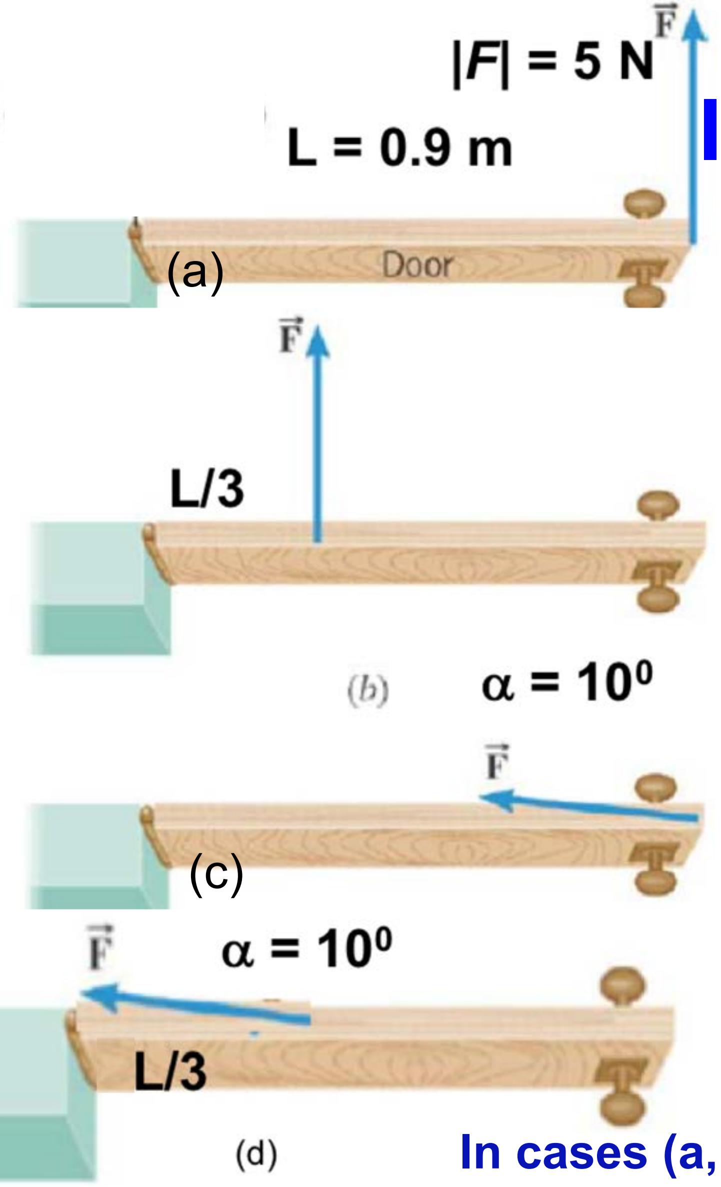
$|\tau_{F0}| = r \cdot F_y = 0 \cdot 5 = 0\ N.m$

0.9m -0.9m 0m

$|\tau| = \frac{2}{3} \cdot 0.9 \cdot 5 =$

$= 3\ Nm$

In cases (a, c) assume the doorknob is at the edge of the door.

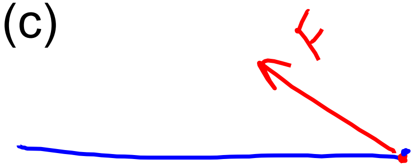


Webassign: L14 Q2

In case (d), relative to the *doorknob* torque is 1. > 0 2. = 0 3. < 0

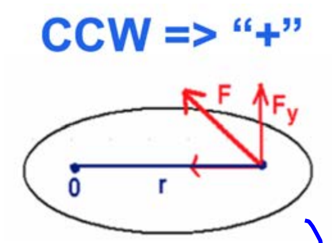
(c)

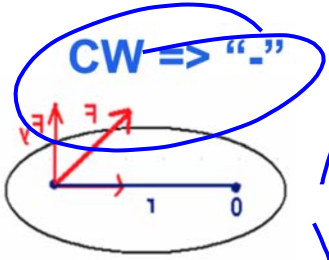
(d)



$$|\vec{\tau}| = r \cdot F_y = 0 \cdot F_y = \phi$$

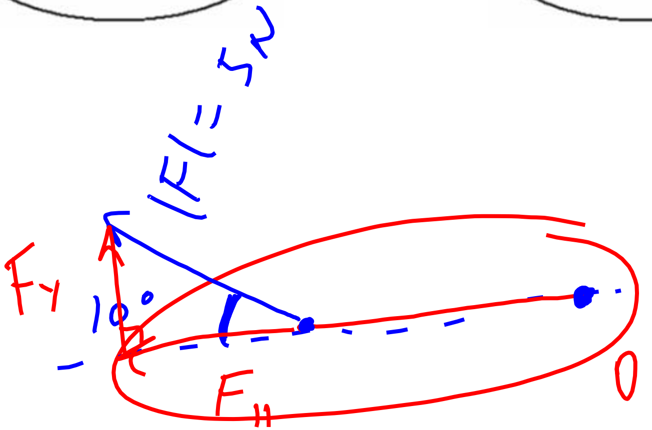
In cases (a, c) assume the doorknob is at the edge of the door.





Webassign: L14 Q2

In case (d), relative to the *doorknob* torque is 1. > 0 2. = 0 3. < 0

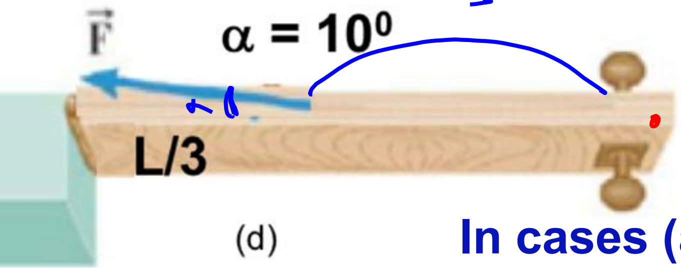


$$|\tau| = r \cdot F_y = \frac{2}{3} \cdot 0.9 \cdot |F| \cdot \sin\alpha =$$

$$= 0.6 \cdot 5 \cdot \sin 10° = .52\ Nm$$

CW ⇒ "−"

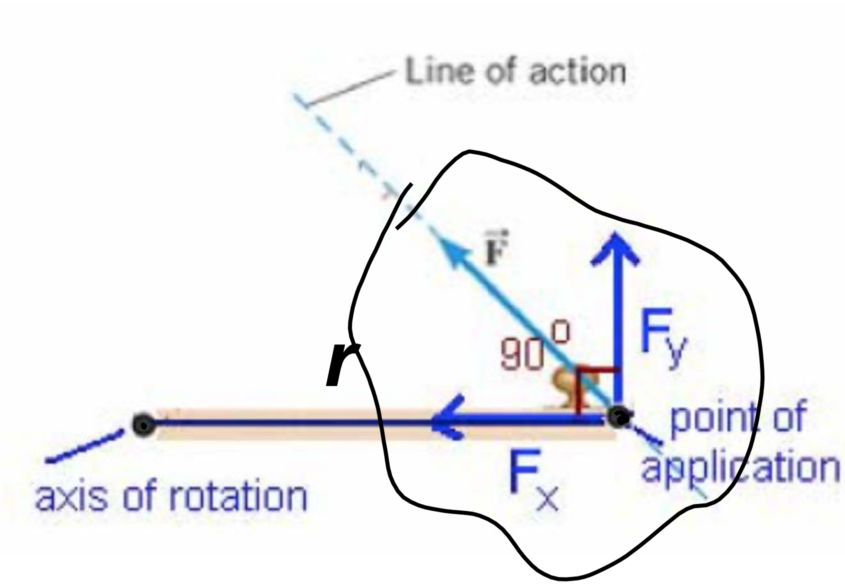
$\frac{2}{3} L$



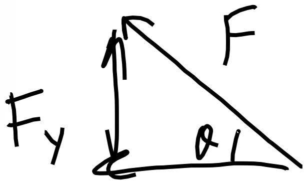
In cases (a, c) assume the doorknob is at the edge of the door.

Definition + geometry =

Torque (a second expression)



$$|\tau| = r \cdot |F_y| = \boxed{r \cdot F \cdot \sin\theta}$$



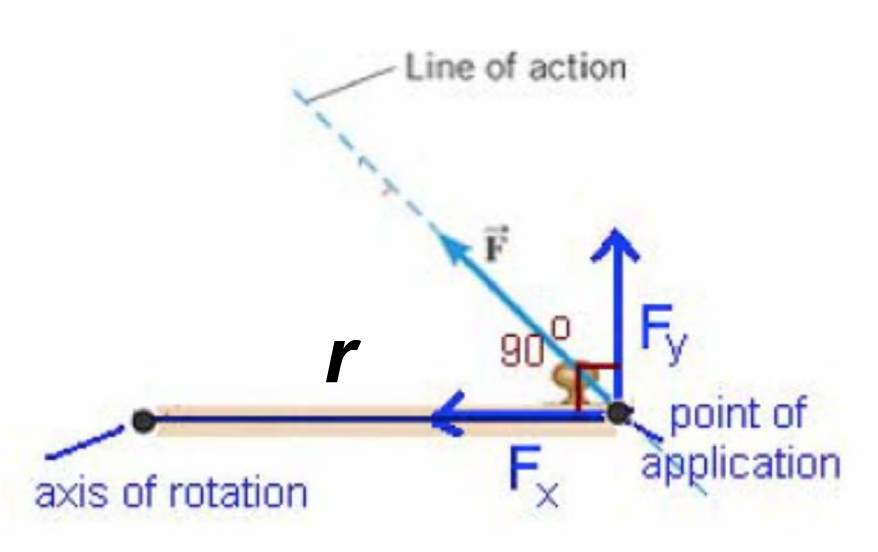
$$|\tau| = r|F_y|$$

The Definition

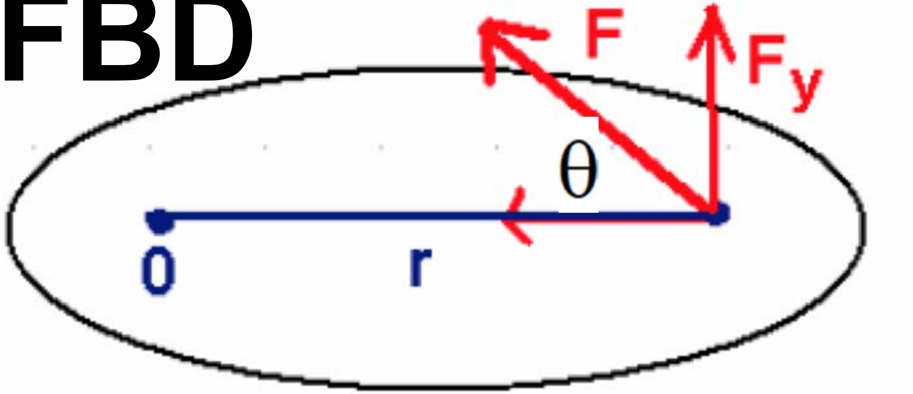
Definition + geometry =

Torque

$$|F_y| = |F| * |\sin(\theta)|$$



FBD



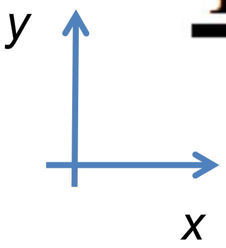
$$|\tau| = \mathbf{r}\,|\mathbf{F_y}|$$

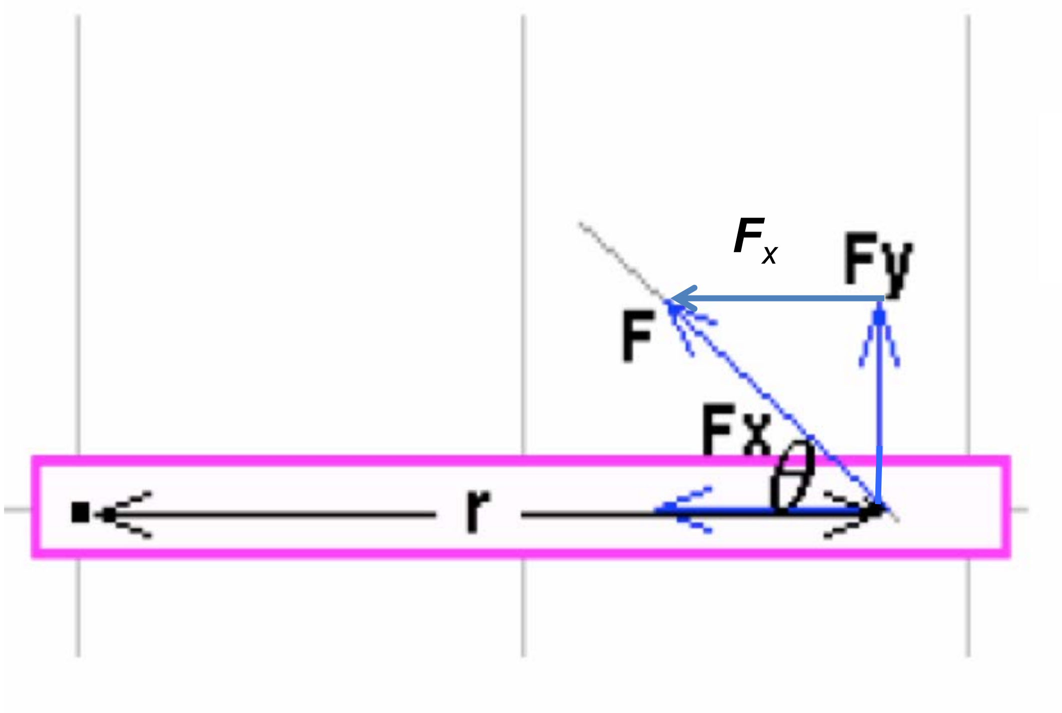
The Definition

$$|\tau| = r\,|F \sin(\theta)|$$

The equivalent expression

Torque





Calculation 1

$$|\tau| = r|F_y|$$

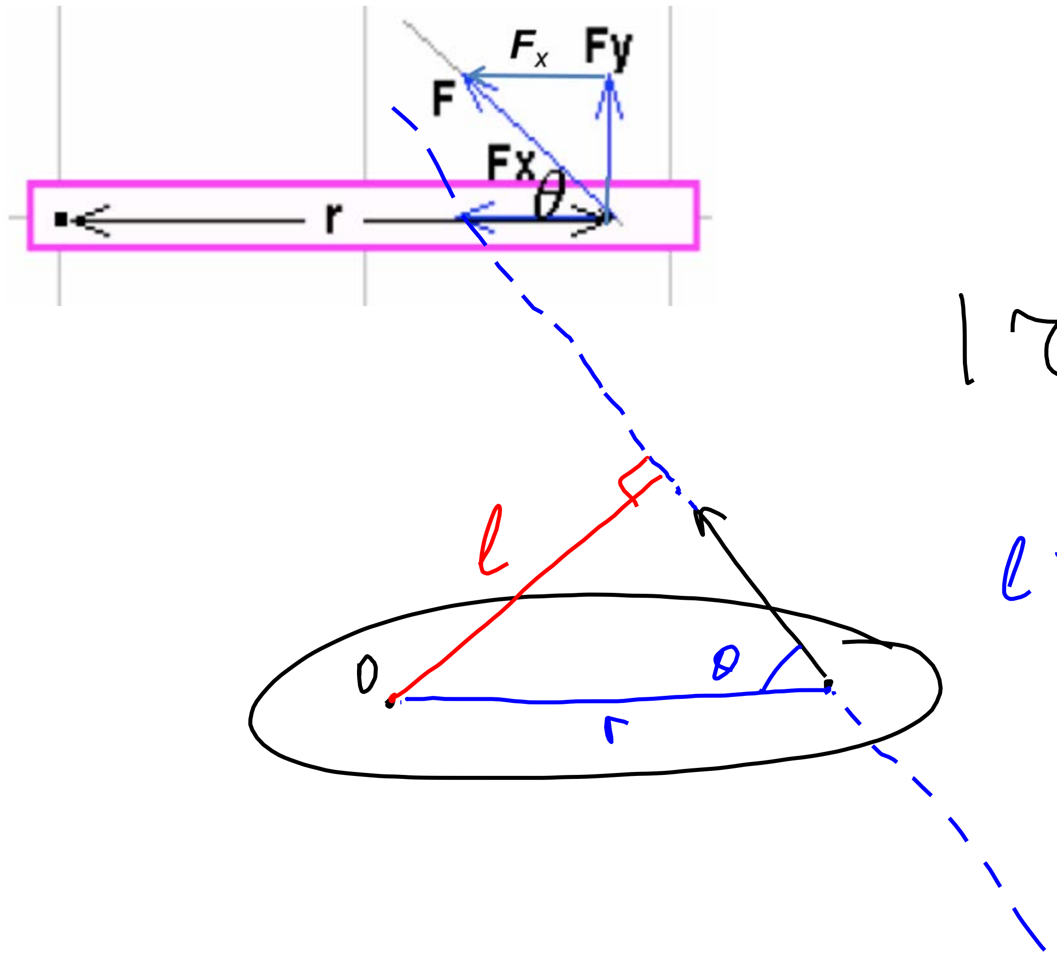
Calculation 2

$$|\tau| = r|F\sin(\theta)|$$

Torque

Definition + geometry =

(a third expression: the most practical)



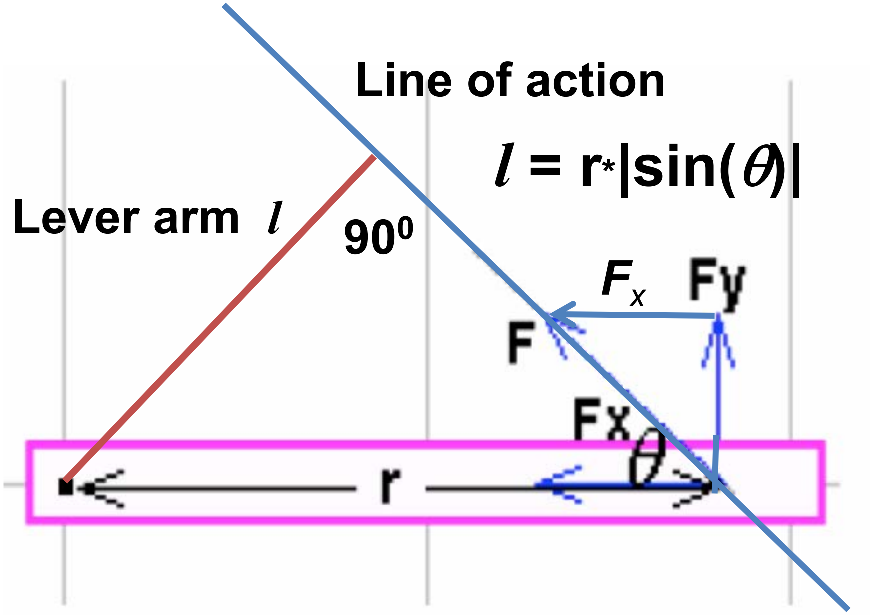
$$|\tau| = r\cdot F_y = r\cdot F\cdot\sin\theta =$$

$$= F\cdot \underbrace{r\cdot\sin\theta} =$$

$$\ell = r\cdot\sin\theta$$

$$= \underbrace{F\cdot\ell}$$

Torque



The length of a Lever Arm equals the shortest distance from the axis of rotation to the line of action.

Definition + geometry =

Calculation 1

$$|\tau| = r|F_y|$$

Calculation 2

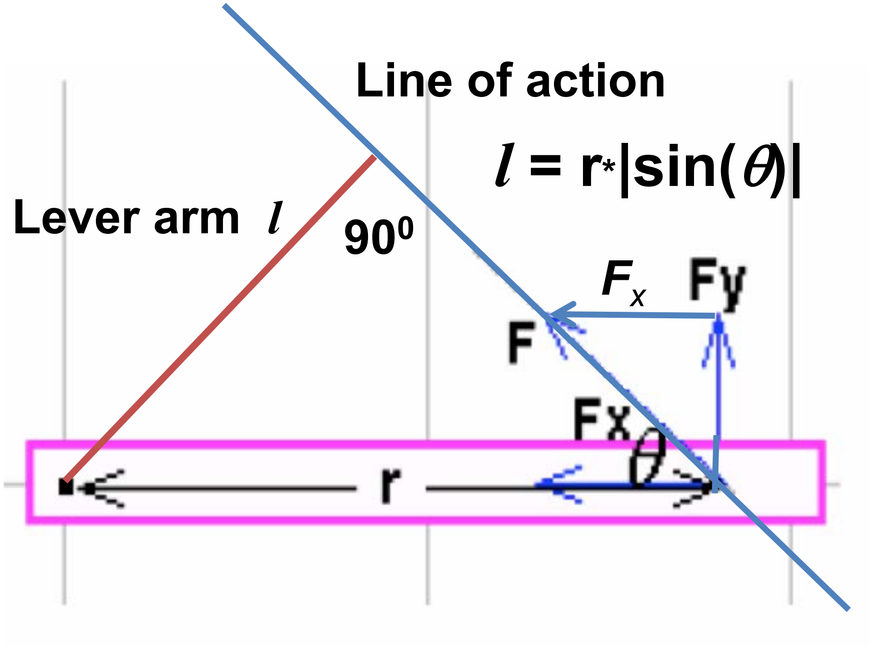
$$|\tau| = r|F\sin(\theta)|$$

Calculation 3

$$\boxed{|\tau| = F\ell}$$

Torque

$$|\tau| = F\ell$$



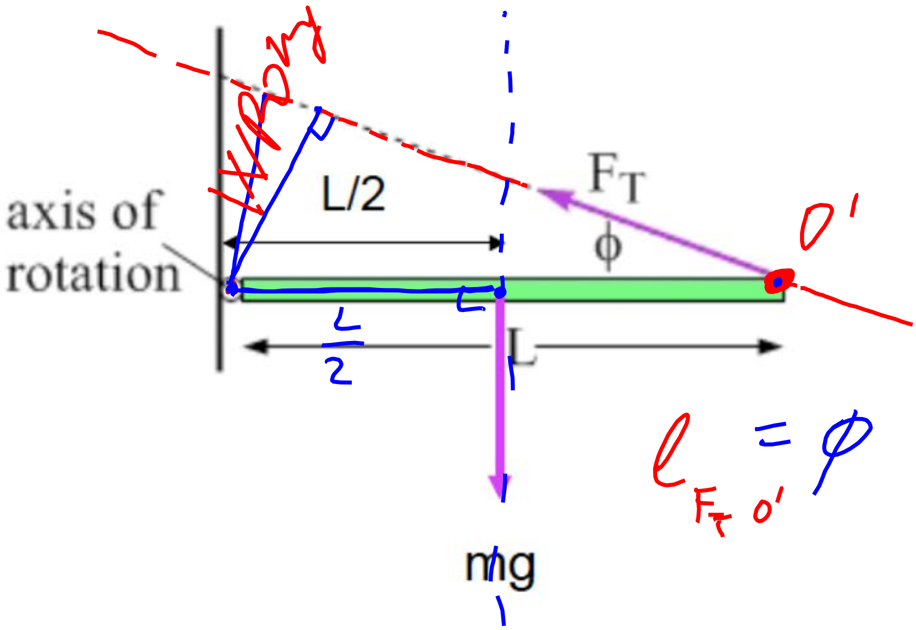
The length of a Lever Arm equals the shortest distance from the axis of rotation to the line of action.

To draw a lever arm:

1. Draw the line of action (extend the force)

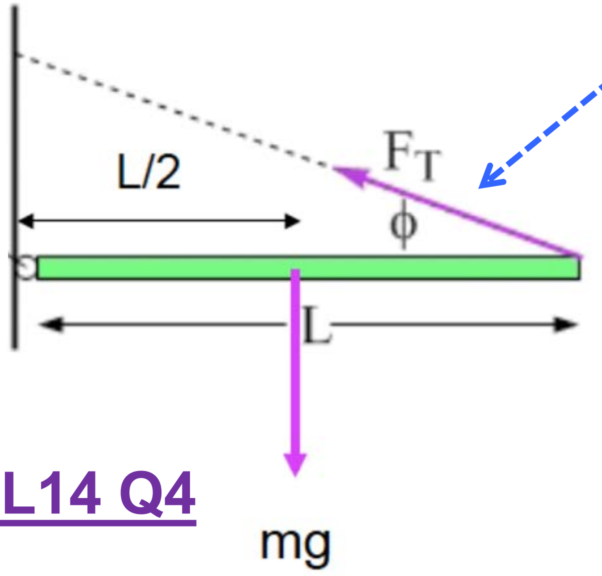
2. Draw the shortest path from the axis to the line of action

To draw a lever arm:
1. Draw the line of action (extend the force)
2. Draw the shortest path from the axis to the line of action



Draw the lever arm for the force of tension relative to the hinge

Webassign: L14 Q3

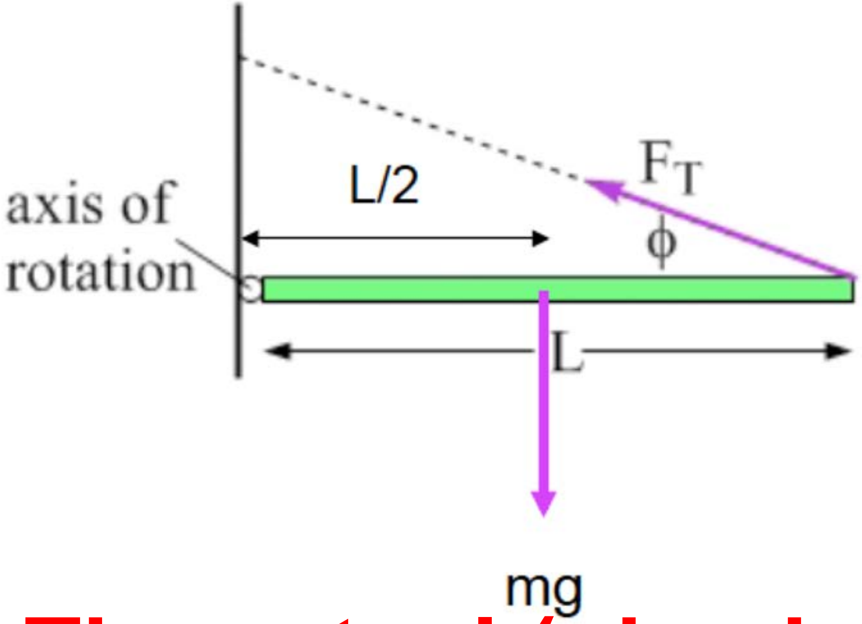


The torque of the *tension* force relative to the left end of the rod is …
1. Negative
2. Zero
3. Positive
4. Not enough information to decide

Webassign: L14 Q4

The torque of the *tension* force relative to the right end of the rod is …
1. Negative
2. Zero
3. Positive
4. Not enough information to decide

“torque” is a shortcut for



“THE TORQUE OF this FORCE REALTIVE TO this AXIS”

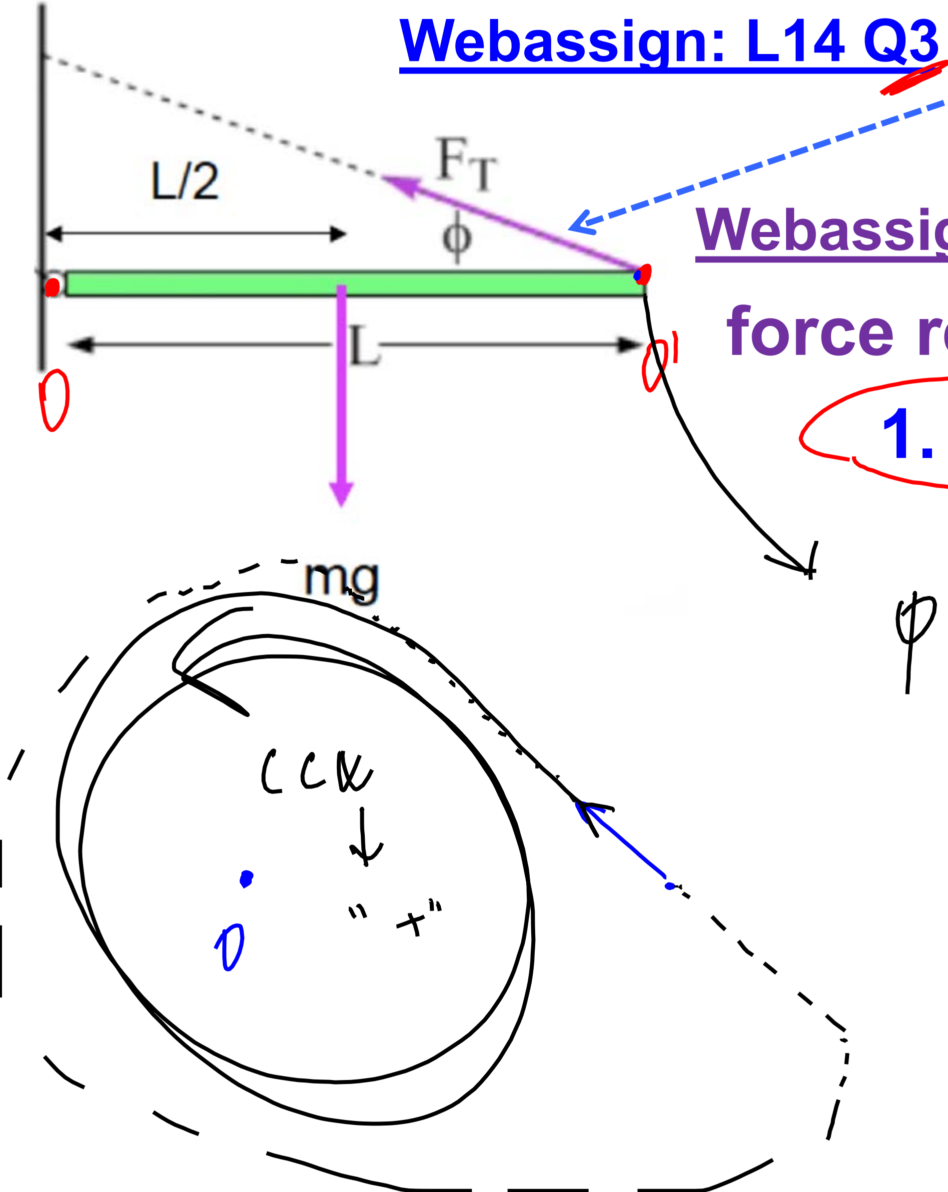
The actual (physical) axis of rotation is not always equal to the abstract (mathematical) axis of rotation.

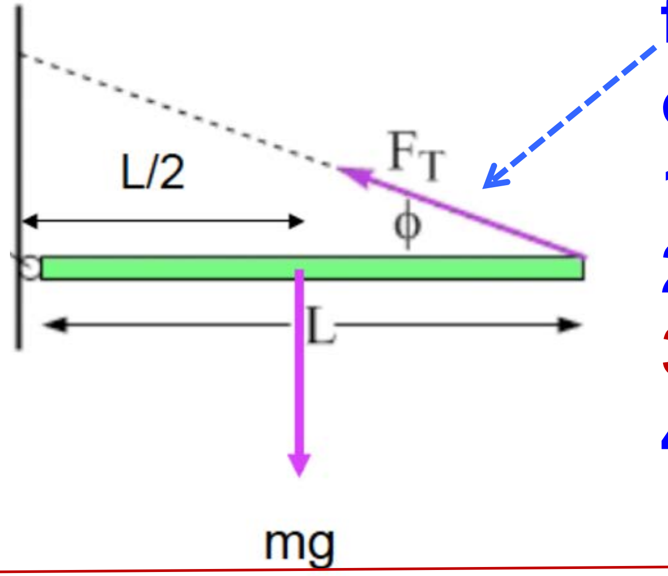
You can choose the axis of ration!

Webassign: L14 Q3 The torque of the *tension* force relative to the left end

Webassign: L14 Q4 The torque of the *tension* force relative to the right

1. Negative 2. Zero 3. Positive





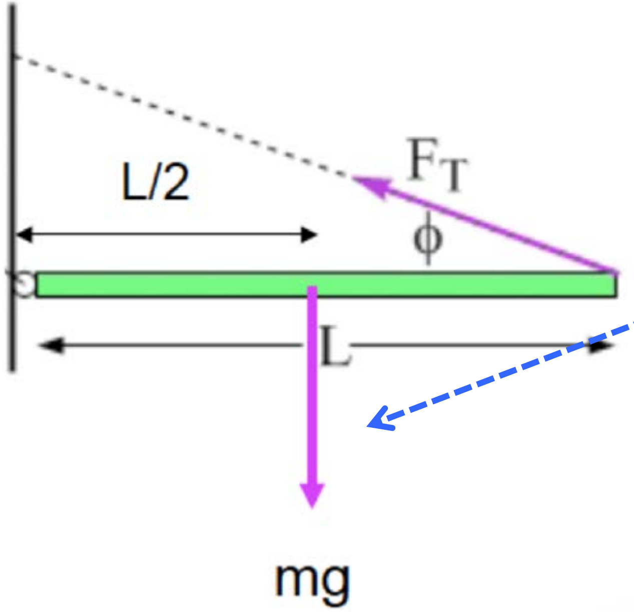
The torque of the ***tension*** force relative to the left end of the rod is …

1. Negative
2. Zero
3. Positive (CCW)
4. Not enough information to decide

The torque of the ***tension*** force relative to the right end of the rod is …

1. Negative
2. Zero
3. Positive
4. Not enough information to decide

The torque of the *force of gravity …*



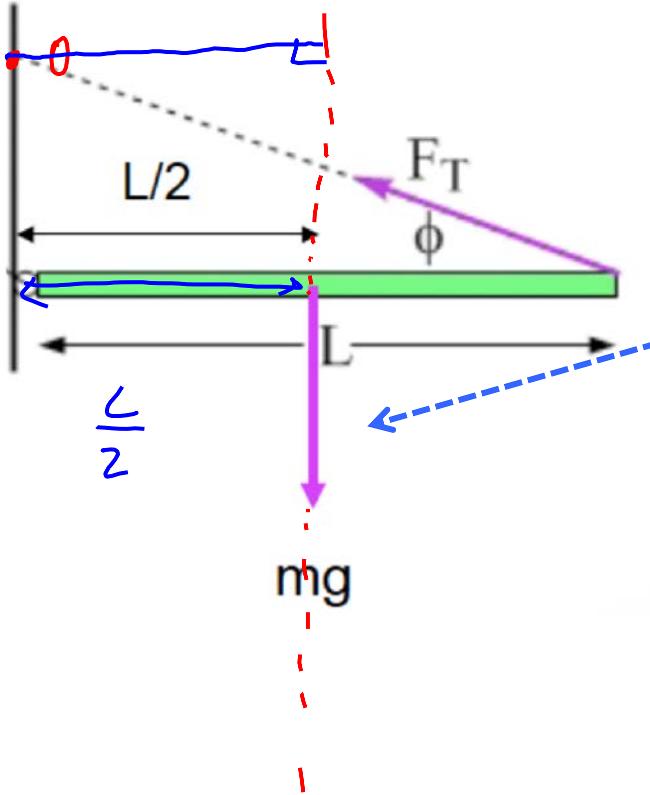
For a solid object the force of gravity is *always* applied to its CofM!

(we will prove this statement later)

A uniform object (with the same mass density everywhere in it) has CofM at its center of symmetry ("middle point")

(we may prove this statement later)

Find the torque of the *force of gravity* relative to …



$$|\tau_{mg,O}| = |mg| \cdot \ell_{mg} = mg\frac{L}{2}$$

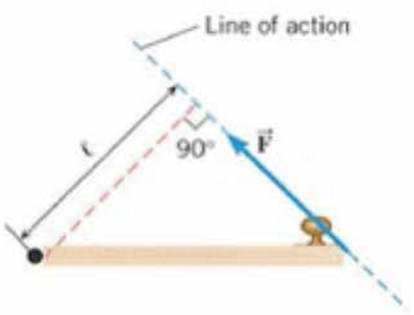
torque of a force

$$|\tau| = F\ell$$ ⟷ **Magnitudes!**

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.

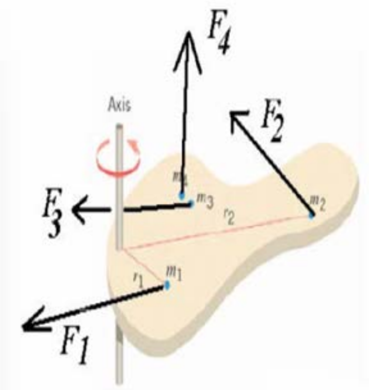
***Actual* values!**
(a.k.a. components)

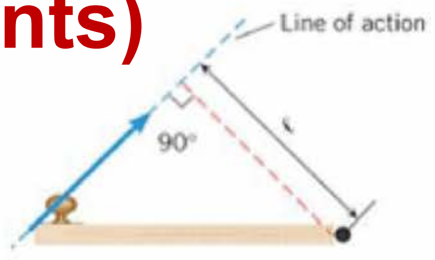
$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \ldots$$



CCW
Positive torque

"out" ⊙



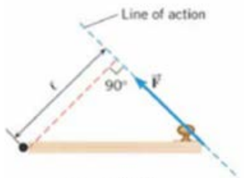


CW
Negative torque

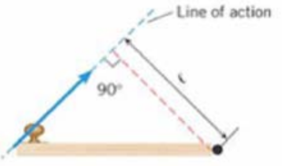
"into" ⊗

torque of a force $|\tau| = F\ell$ ⟷ **Magnitudes!**

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.



CCW
Positive torque



CW
Negative torque

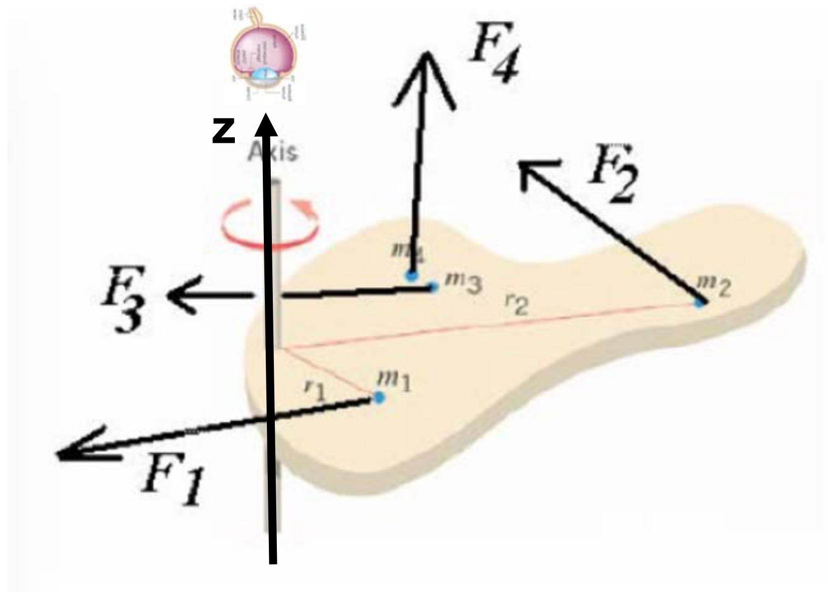
"out" ⊙ **"into"** ⊗

***Actual* values!**
(a.k.a. components)

Net $\vec{\text{T}}$orque = $\vec{\text{T}}$orque1 + $\vec{\text{T}}$orque2 + …

$$\vec{\tau}_{NET} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots$$

$$\tau_{net_z} = \tau_{1_z} + \tau_{2_z} + \ldots$$



Equilibrium: TBE

$$\vec{\tau}_{Net} = 0$$

$$\tau_{1z} + \tau_{2z} + \ldots = 0$$

Actual values! (a.k.a. z-components)

OR

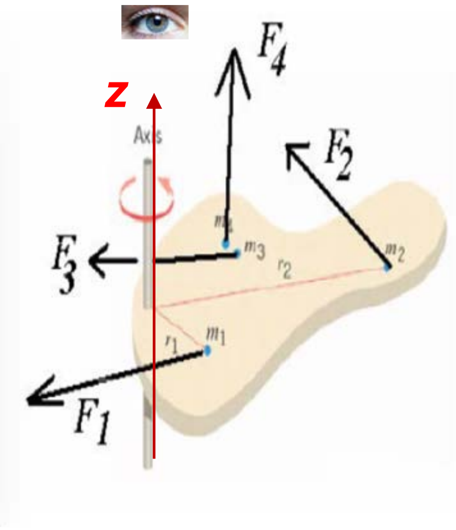
Magnitudes! (use: CCW is +; CW is -)

$$\tau_{1ccw} + \tau_{2ccw} + \ldots - \tau_{1cw} - \tau_{2cw} - \ldots = 0$$

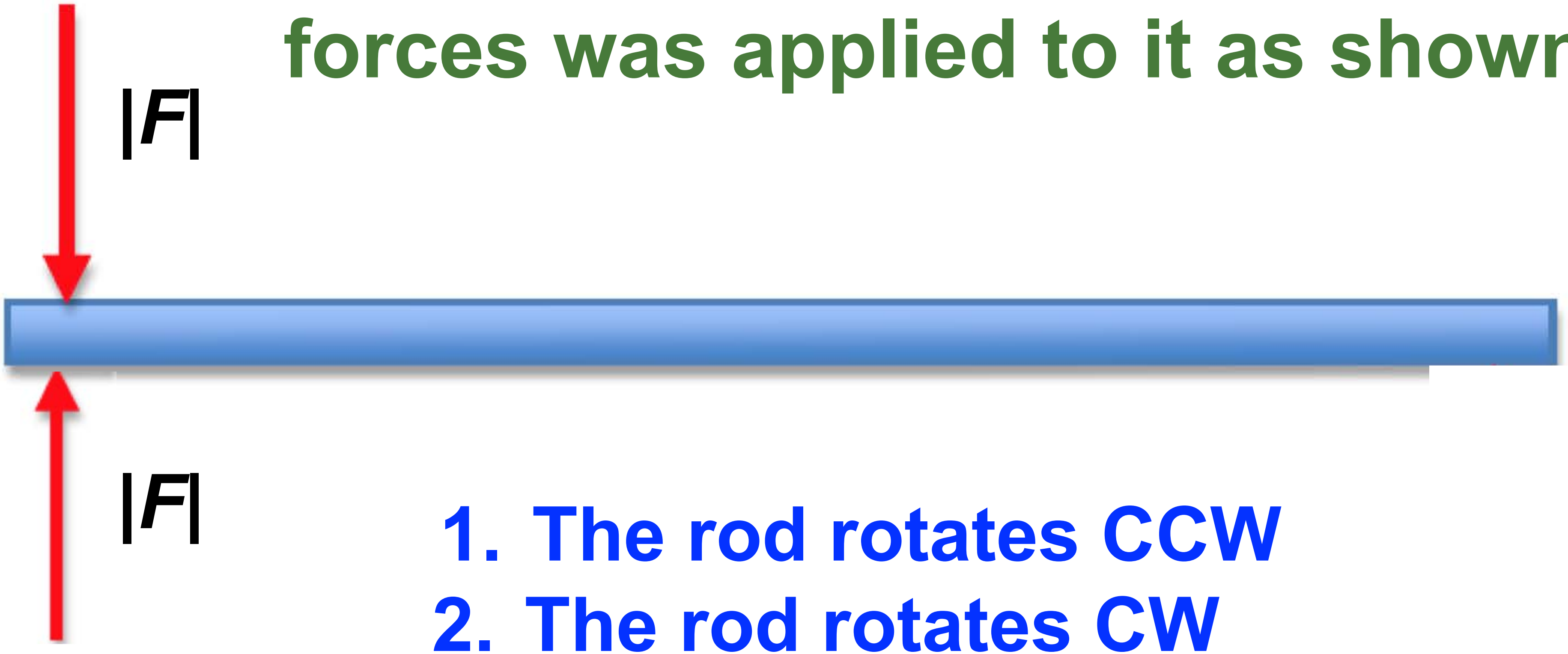
OR

$$\tau_{1ccw} + \tau_{2ccw} + \ldots = \tau_{1cw} + \tau_{2cw} + \ldots$$

Magnitudes!



The rod was at rest when a pair of forces was applied to it as shown.

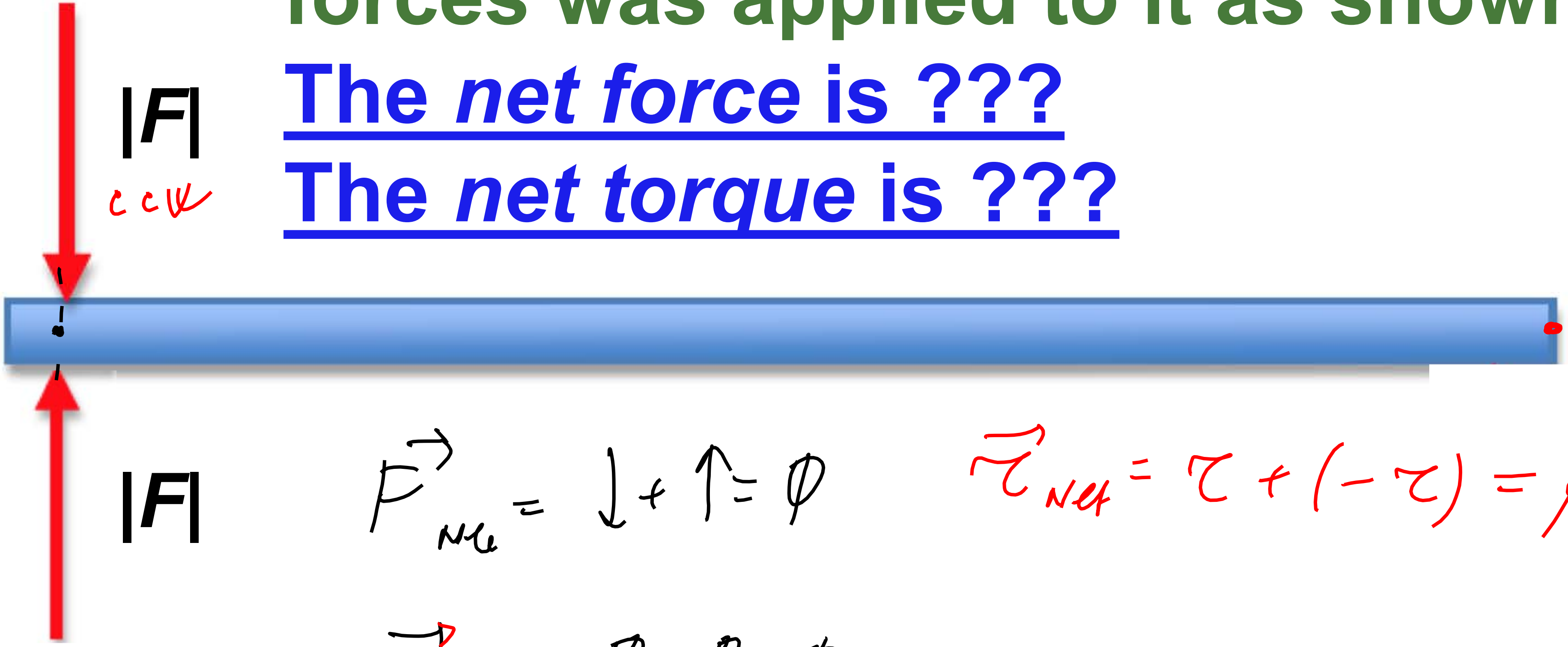


1. The rod rotates CCW
2. The rod rotates CW
3. None of the above
4. All of the above

The rod was at rest when a pair of forces was applied to it as shown.

The *net force* is ???

The *net torque* is ???

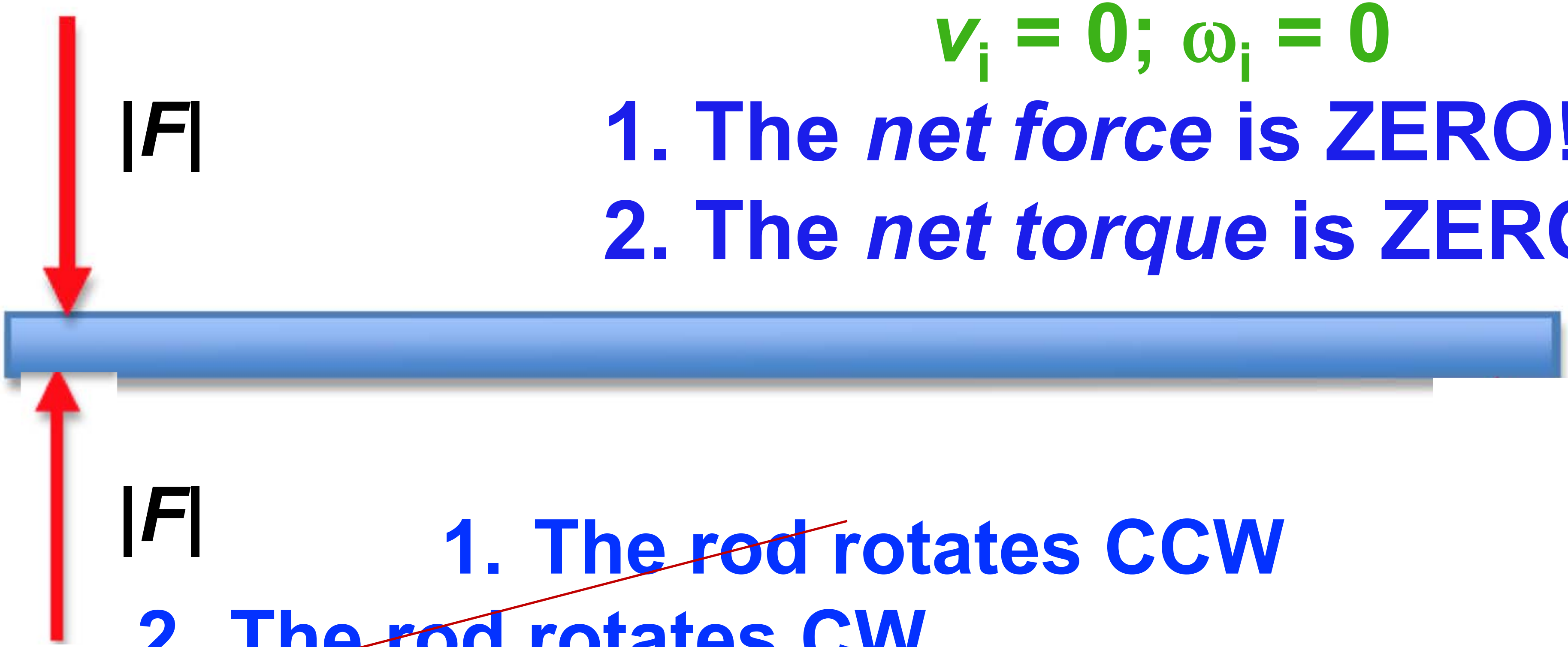


$\vec{F}_{Net} = \downarrow + \uparrow = \phi$

$\vec{\tau}_{Net} = \tau + (-\tau) = \phi$

$\vec{\tau}_{Net} = 0 + 0 = \phi$

Equilibrium



$v_i = 0; \omega_i = 0$

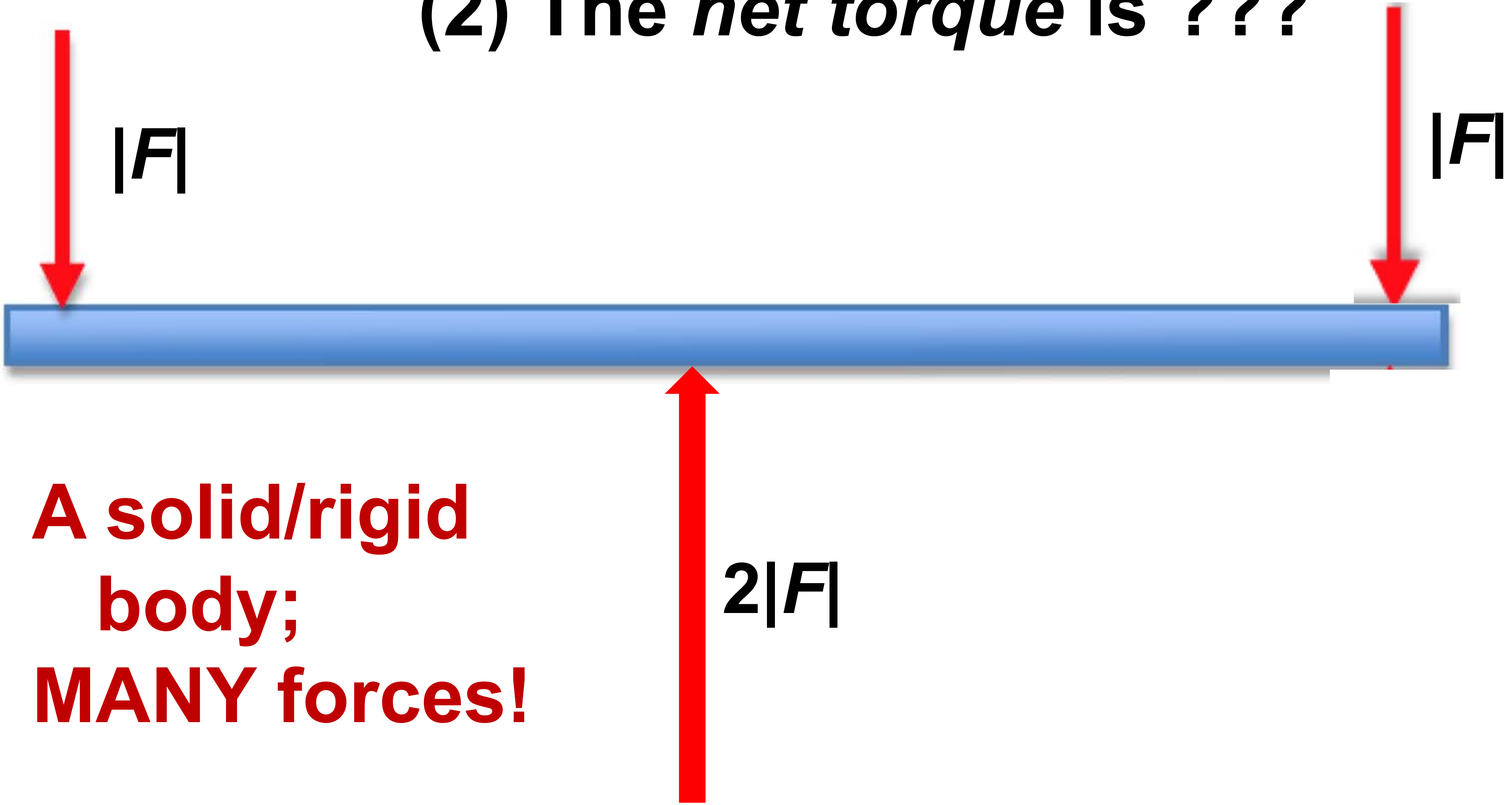
1. The *net force* is ZERO!
2. The *net torque* is ZERO.

1. ~~The rod rotates CCW~~
2. ~~The rod rotates CW~~
3. None of the above = it remains at rest
4. ~~All of the above~~

$v_i = 0; \omega_i = 0$

(1) The *net force* is ???

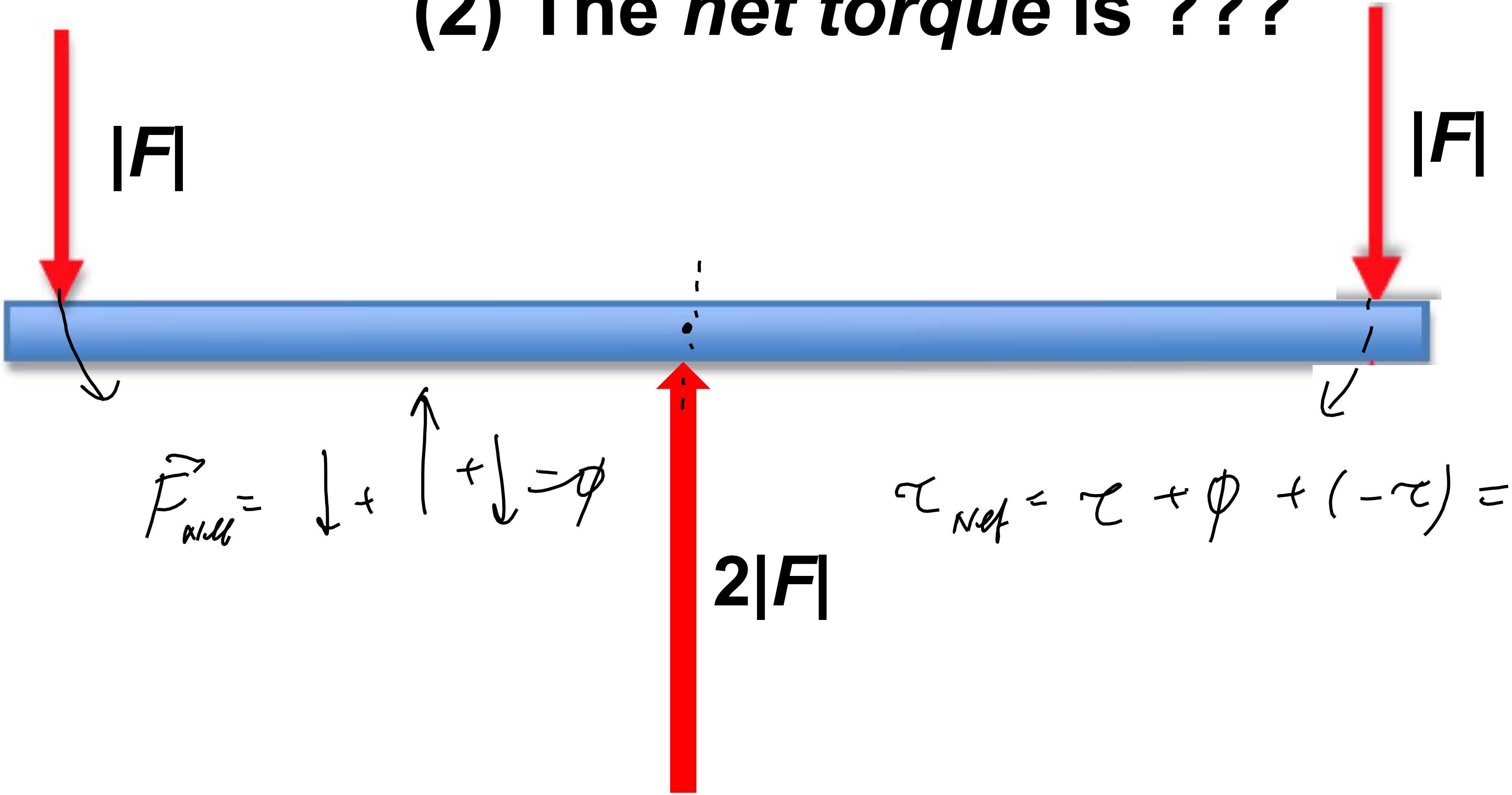
(2) The *net torque* is ???



A solid/rigid body;
MANY forces!

$v_i = 0$; $\omega_i = 0$

(1) The *net force* is ???
(2) The *net torque* is ???



$\vec{F}_{net} = \downarrow + \uparrow + \downarrow = \phi$

$\tau_{net} = \tau + \phi + (-\tau) = \phi$

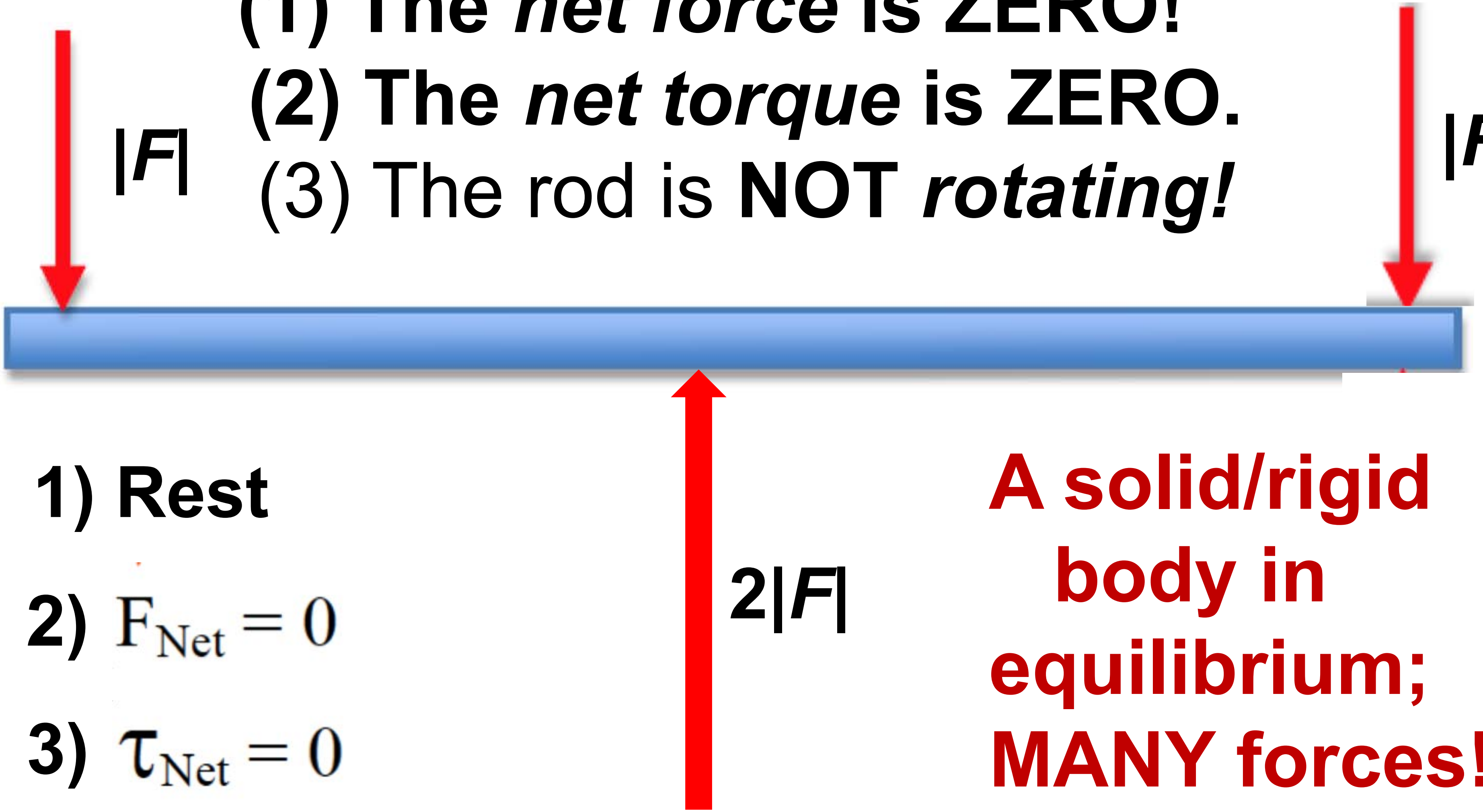
Equilibrium

$v = 0; \omega = 0$

(1) The *net force* is ZERO!

(2) The *net torque* is ZERO.

(3) The rod is **NOT *rotating!***



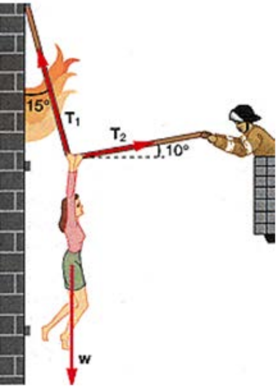
1) Rest

2) $F_{Net} = 0$

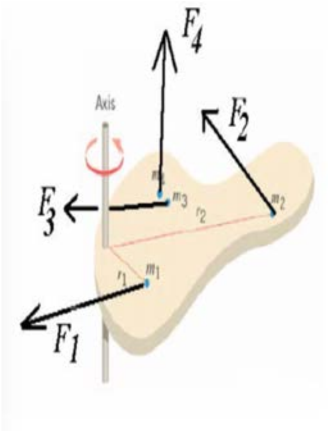
3) $\tau_{Net} = 0$

A solid/rigid body in equilibrium; MANY forces!

Rigid Objects in Equilibrium



A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration and not moving.



In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

In general, x and y components of the forces and z component of the torques have to be considered:

For a small (point like) object <u>and</u> for a large solid object

$$\boxed{\sum F_x = 0 \qquad \sum F_y = 0}$$

N II L
(balance of forces equation)
Net Force = 0

$v = 0,\ a = 0$

$$\sum \tau = 0$$

For a large solid object

a new condition
(balance of torques equation)
Net Torque = 0

$\omega = 0,\ \alpha = 0$

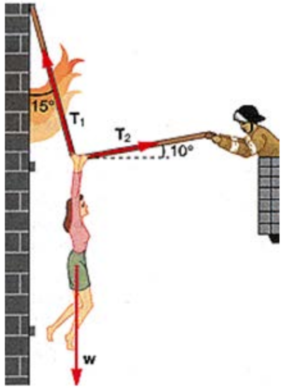
Equilibrium

a.k.a. "rest"

Equilibrium is the absence of ANY motion: translational as well as rotational.

http://dictionary.reference.com/browse/axes

Conditions for equilibrium:



1) an object is initially at rest

FBE

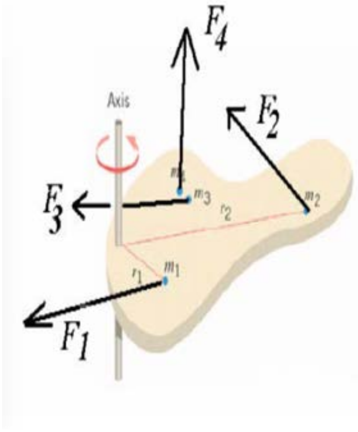
2) $F_{Net} = 0$ (no translational motion!)

(relative to x and y axes)

TBE

3) $\tau_{Net} = 0$ (no rotational motion!)

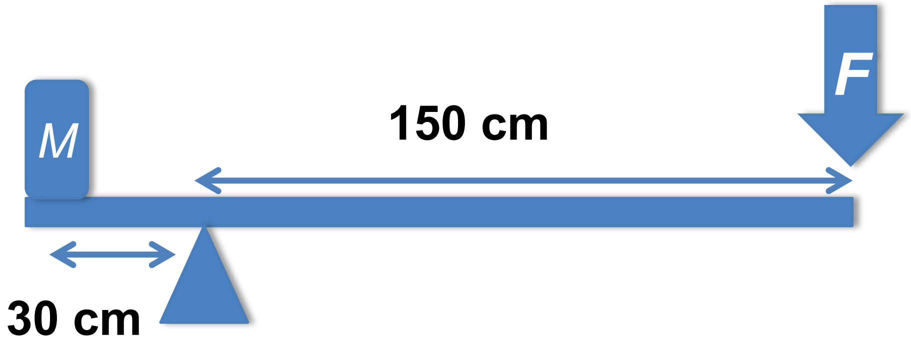
(relative to all *axes* of rotation)



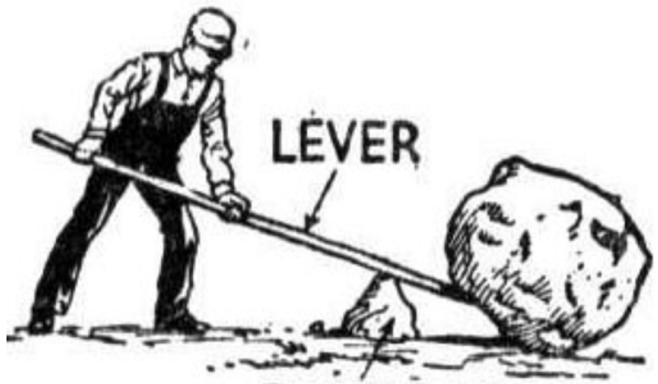
General strategy for using TBE.

1. Picture
2. Convert picture into a diagram: FBD
3. Convert FBD into torque-diagram by SETTING the axis of rotation
4. Write the *actual* value (a.k.a. a component) for a torque of each force relative to *the same* axis
5. Set TBE and solve it
6. If needed, select another axis or use FBE

Equilibrium: examples



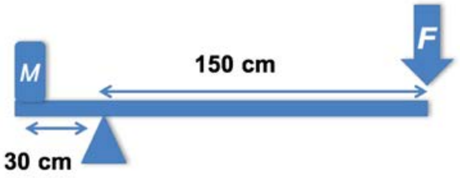
If the maximum weight you can move is 60 lb. (about 27 kg), find the maximum weight you can lift using a lever. Assume the lever has no mass.



Man lifting a stone with a lever

"Give me a lever long enough and a fulcrum on which to place it, and I shall move the world". (~260 BC)

https://en.wikiquote.org/wiki/Archimedes



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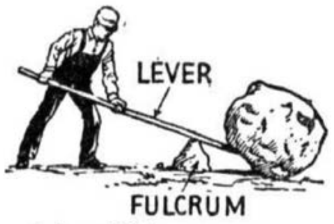

Man lifting a stone with a lever

"Give me a lever long enough and a fulcrum on which to place it, and I shall move the world".
https://en.wikiquote.org/wiki/Archimedes

$$\boxed{\tau = F\ell}$$

CCW => "+" CW => "-"

$$\vec{\tau}_1 + \vec{\tau}_2 + \ldots = 0$$

1. Picture
2. Convert picture into a diagram: FBD
3. Convert FBD into torque-diagram by **SETTING the axis of rotation**
4. Write the actual value (a.k.a. a component) for torque of each force relative to the same axis
5. Set TBE and solve it
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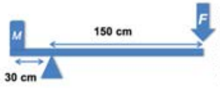

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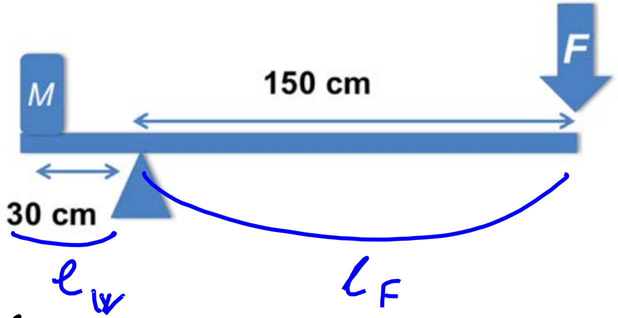
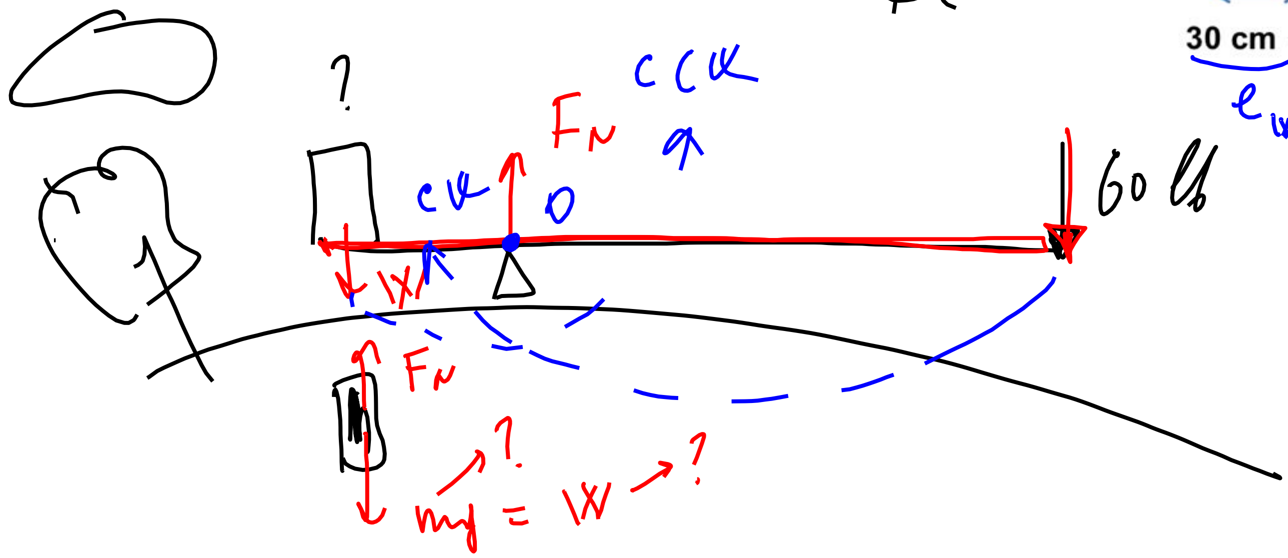


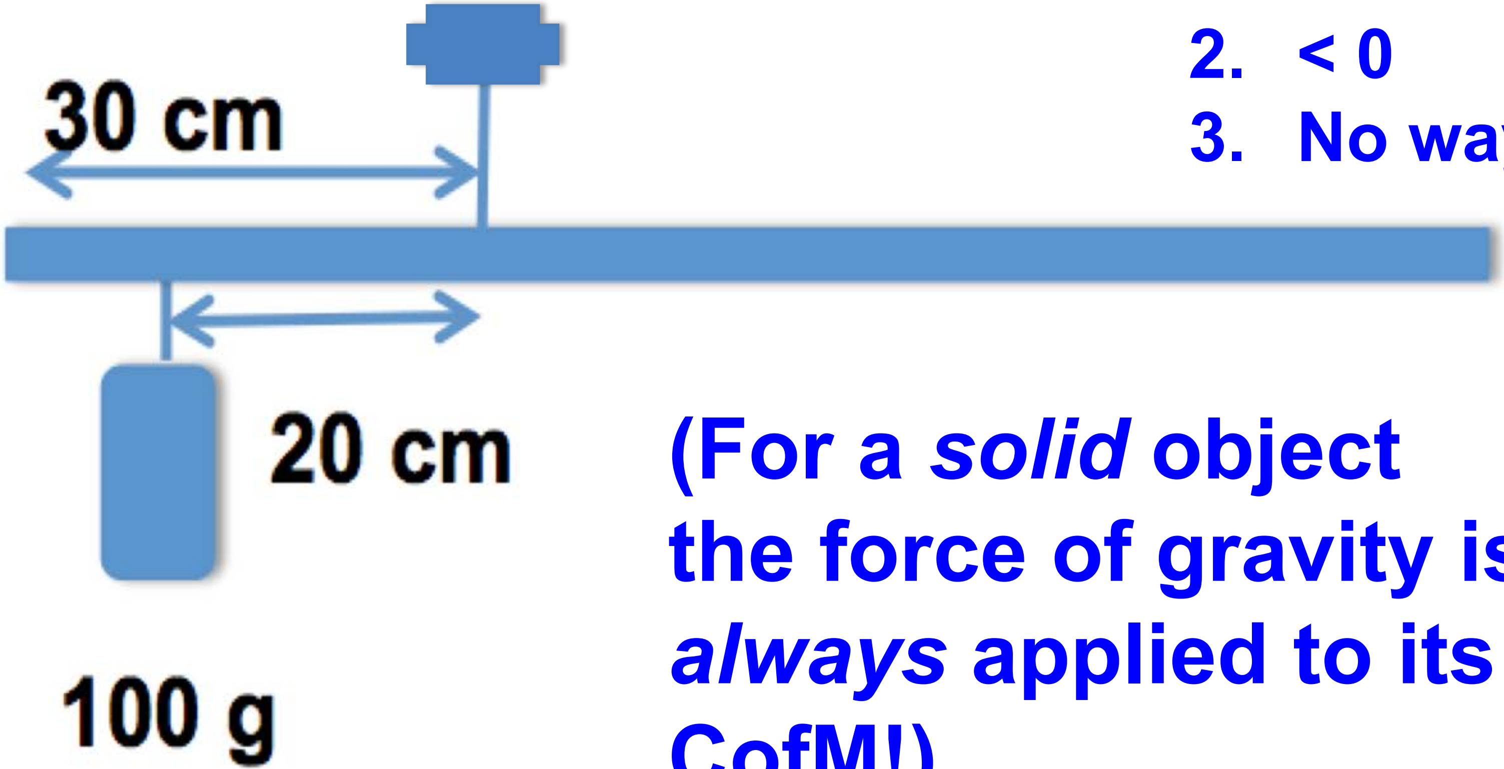


TBE: $\tau_{net} = \emptyset$

$W \cdot 30\cancel{cm} - 60lb \cdot 150\cancel{cm} = 0$

$W = \dfrac{60 \cdot 150}{30} = 300\ lb$

Find the *mass* of the meter stick.



Webassign: L14 Q5

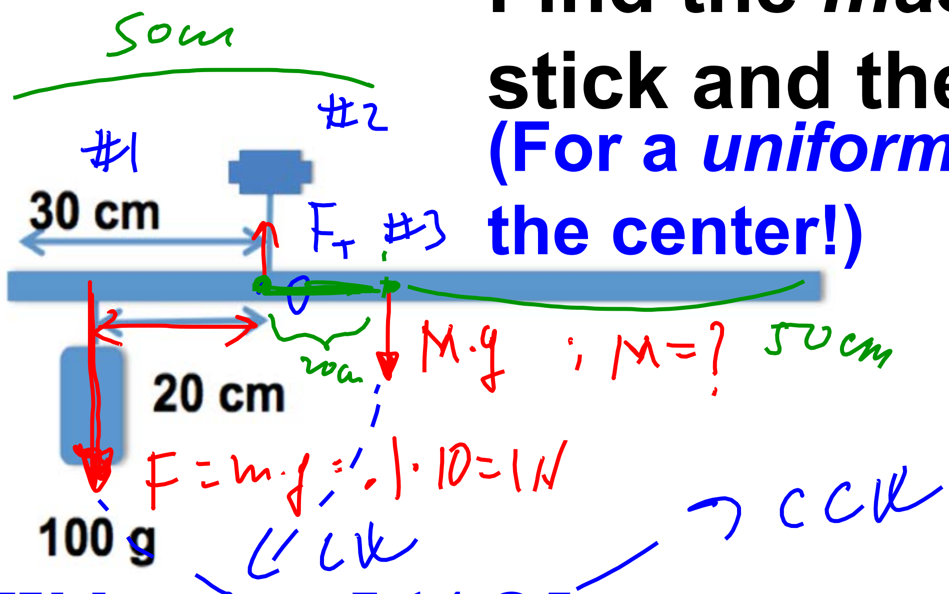
Gravity generates torque, which is

1. > 0
2. < 0
3. No way to say

(For a *solid* object the force of gravity is *always* applied to its CofM!)

Find the *mass* of the meter stick and the tension in the string.

(For a *uniform* solid object the CofM is always at the center!)



$\tau_{net} = 0$; $\tau_1 = +1 \cdot 20\text{cm}$; 0

$\tau_2 = \phi$

$\tau_3 = -\cdot M \cdot g \cdot (20\text{cm})$

$1 \cdot 20 - Mg \cdot 20 = 0$

$Mg = 1\ N$

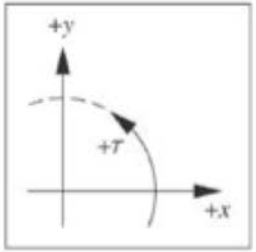
$M = 100\ g$

Webassign: L14 Q5

Gravity generates torque, which is

1. > 0
2. < 0
3. No way to say

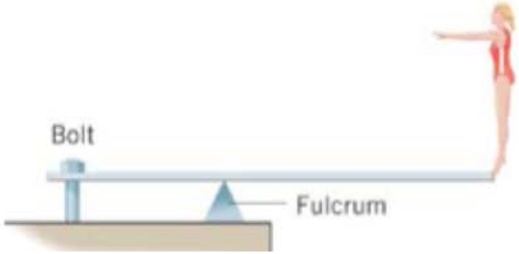
A Diving Board



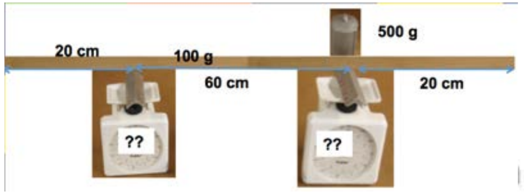
A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m.

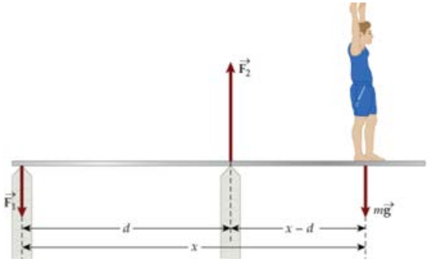
The board has the mass of 12 kg and is supported by a fulcrum 1.40 m away from the left end.

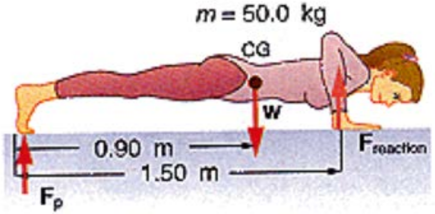
Find the forces that the bolt and the fulcrum exert on the board.

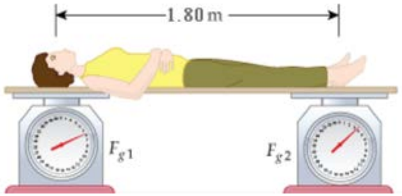


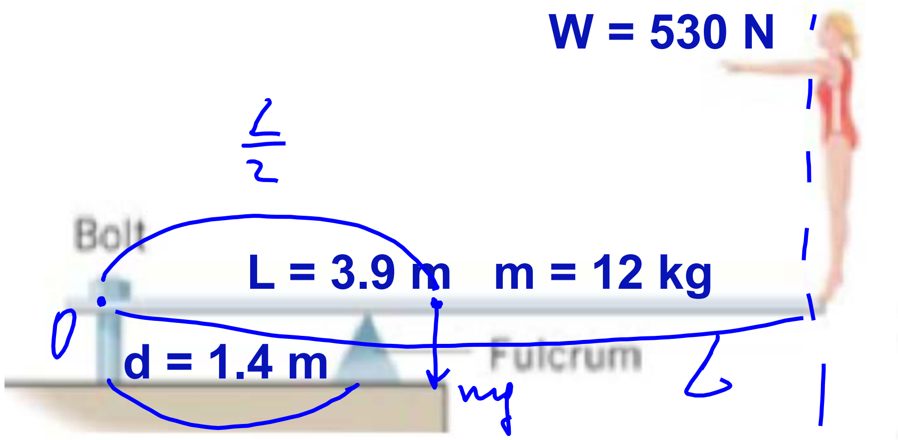
(Note: the same strategy is used for ANY problem with a board placed on two supports, f.e.)







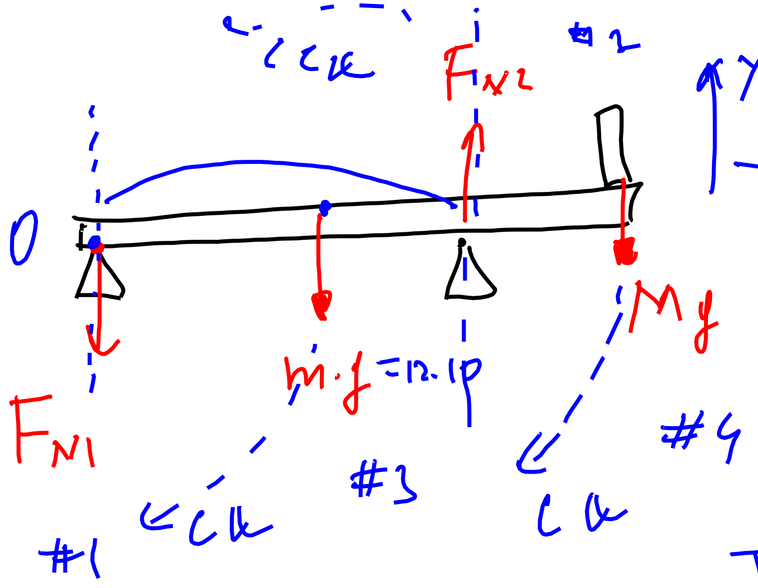




Webassign: L14 Q6

The direction of the force acting *on the board from the bolt* is…

1. Left
2. Right
3. Up
4. Down
5. Into the page
6. Out of the page
7. It does not exist



FBE:

$-F_{N1} - 120 + F_{N2} - 530 = 0$

$\tau_1 = 0$

$\tau_2 = + F_{N2} \cdot 1.4$

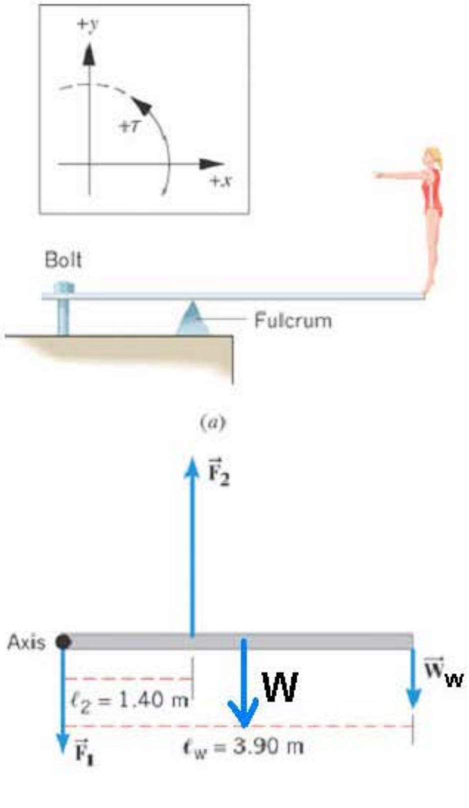
$\tau_3 = - 120 \cdot \frac{3.9}{2}$

$\tau_4 = - 530 \cdot 3.9$

TBE

$0 + F_{N2} \cdot 1.4 - 120 \cdot \frac{3.9}{2} - 530 \cdot 3.9 = 0$

A Diving Board



(b) Free-body diagram of the diving board

$W_W = 530$ N $\quad l_W = 3.9$ m

$M = 12$ kg $\quad (W = 12*9.8 = 117.6$ N$)$,

$l_2 = 1.4$ m

$F_1 = ? \qquad F_2 = ?$

From the balance of forces condition:

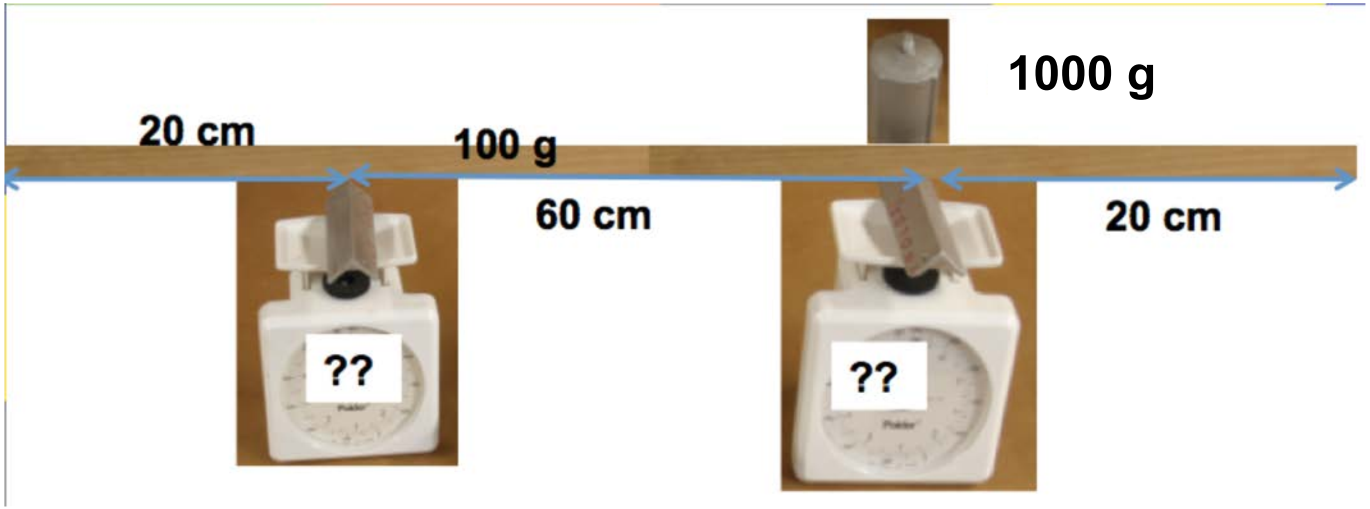
$$F_2 = W + F_1 + W_W$$

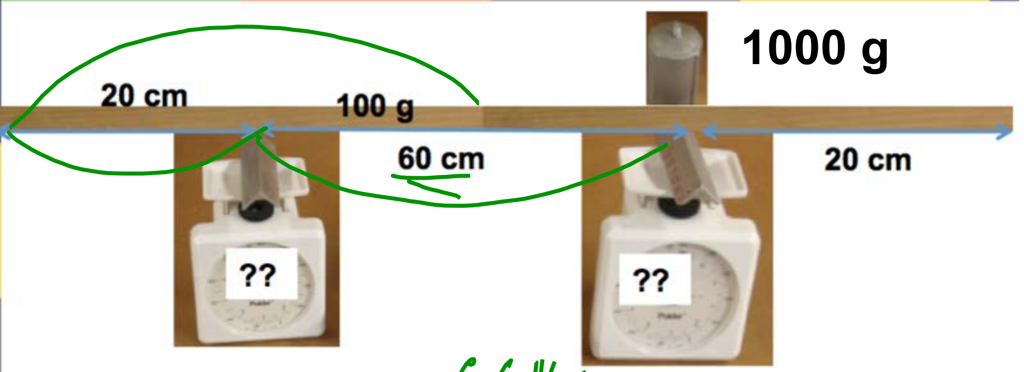
From the balance of torques condition
(written relative to the bolt):

$$\cancel{F_1*0} + F_2*l_2 - W*(l_W/2) - W*l_W = 0$$

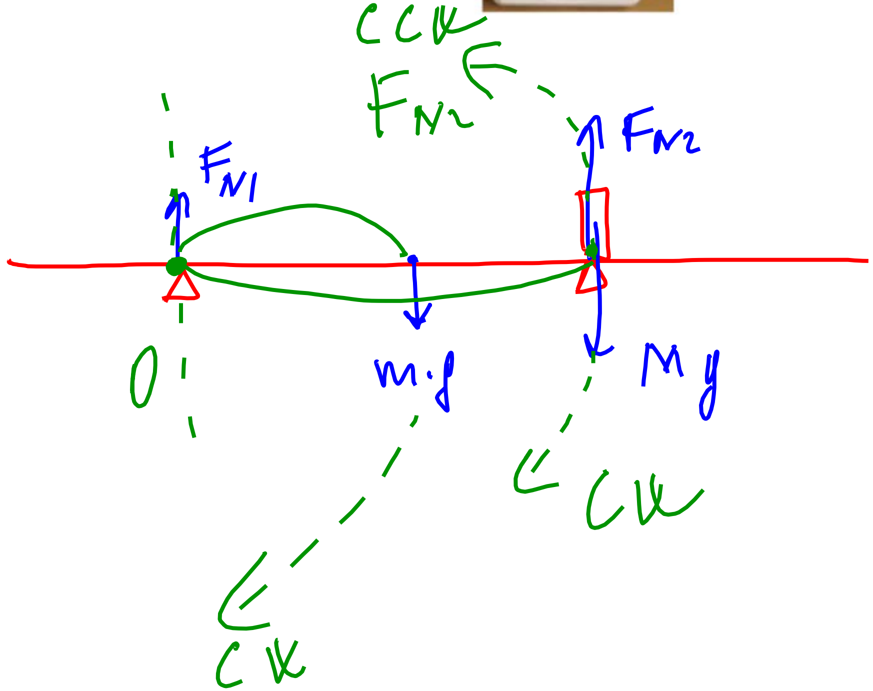
The second equation gives as F_2 and then the first equation gives F_1.

You place a board on two scales and then you place a 1000 g weight on the board. Calculate what reading does each scale show for different positions of the weight.

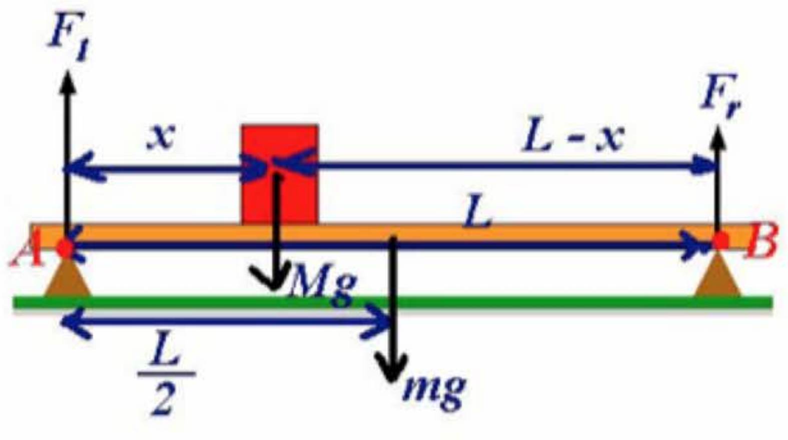




You place a 1000 g weight on a 1 m long board. Calculate what reading does each scale show for different positions of the weight.



$$\text{TBE: } 0 - mg \cdot \underline{\underline{30\,cm}} - Mg \cdot 60\,cm + F_{N2} \cdot 60\,cm = 0$$



The force from the left support increases and the force from the right support decreases

Let's prove it mathematically.

Set the axis at the point A: $\sum\tau = 0$

$Mgx + mgL/2 = F_rL$ or $F_r = \{Mgx + \frac{1}{2}mgL\}/L$ $x\downarrow \Rightarrow F_r\downarrow$

Set the axis at the point B: $\sum\tau = 0$

$mgL/2 + Mg(L - x) = F_lL$ or $F_l = \{\frac{1}{2}mgL + Mg(L - x)\}/L$

$x\downarrow \Rightarrow F_l\uparrow$