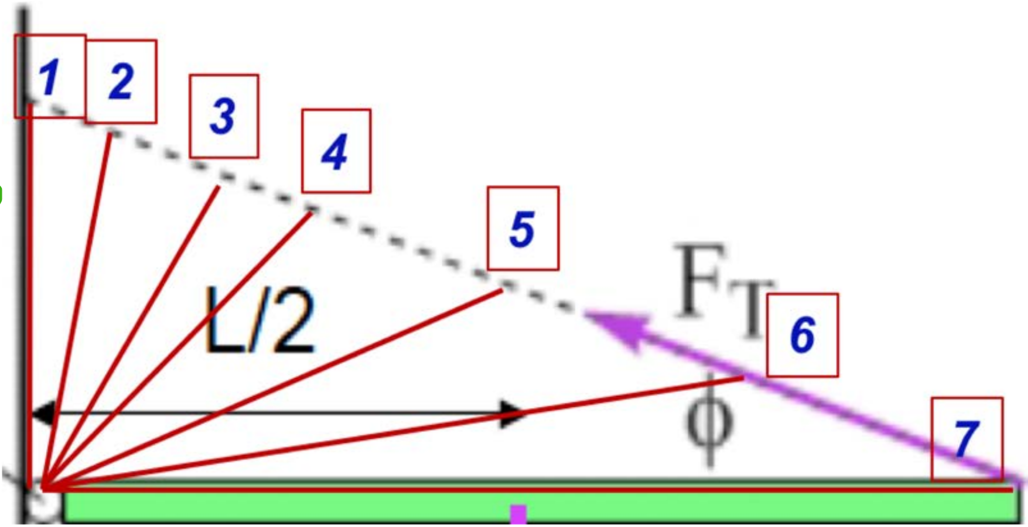
Lab 7 is in SCI 136 (!)

Please, login into webassing, LectureMCQ_L15 (PY105) and answer question 1 (but ONLY Q1!).



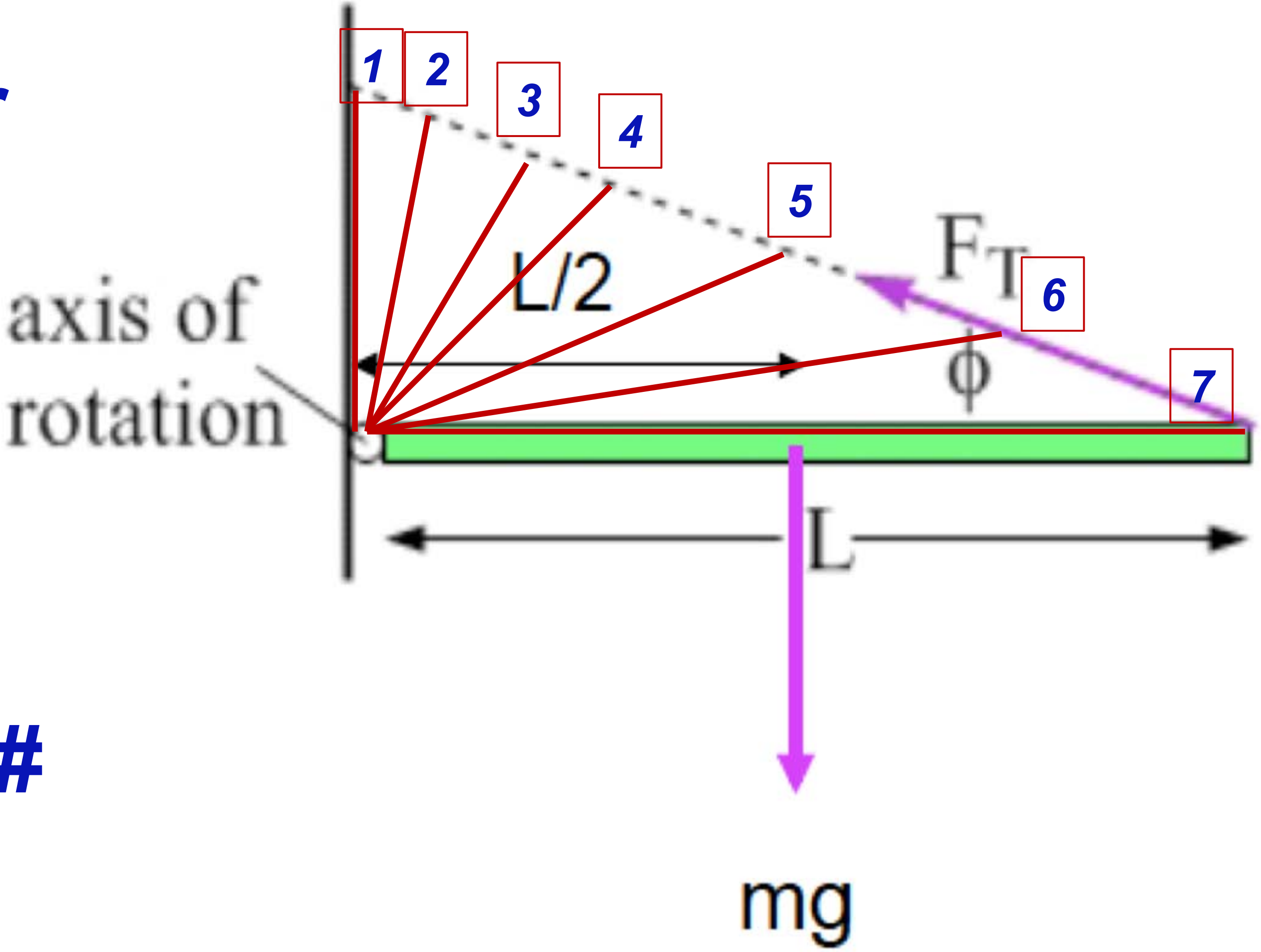
Pleas sign in using the sign-in sheets on the bench. Thank you

Note: exam room change:
Exams 2, 3 take place in STO B50

Good morning!

The lever-arm for the torque of the tension force relative to the hinge is represented by the red segment #

1. 2. 3. …

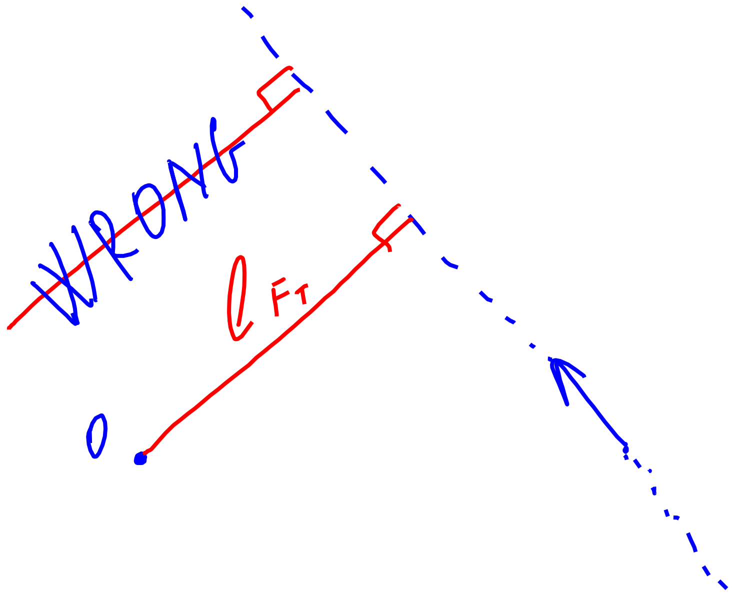


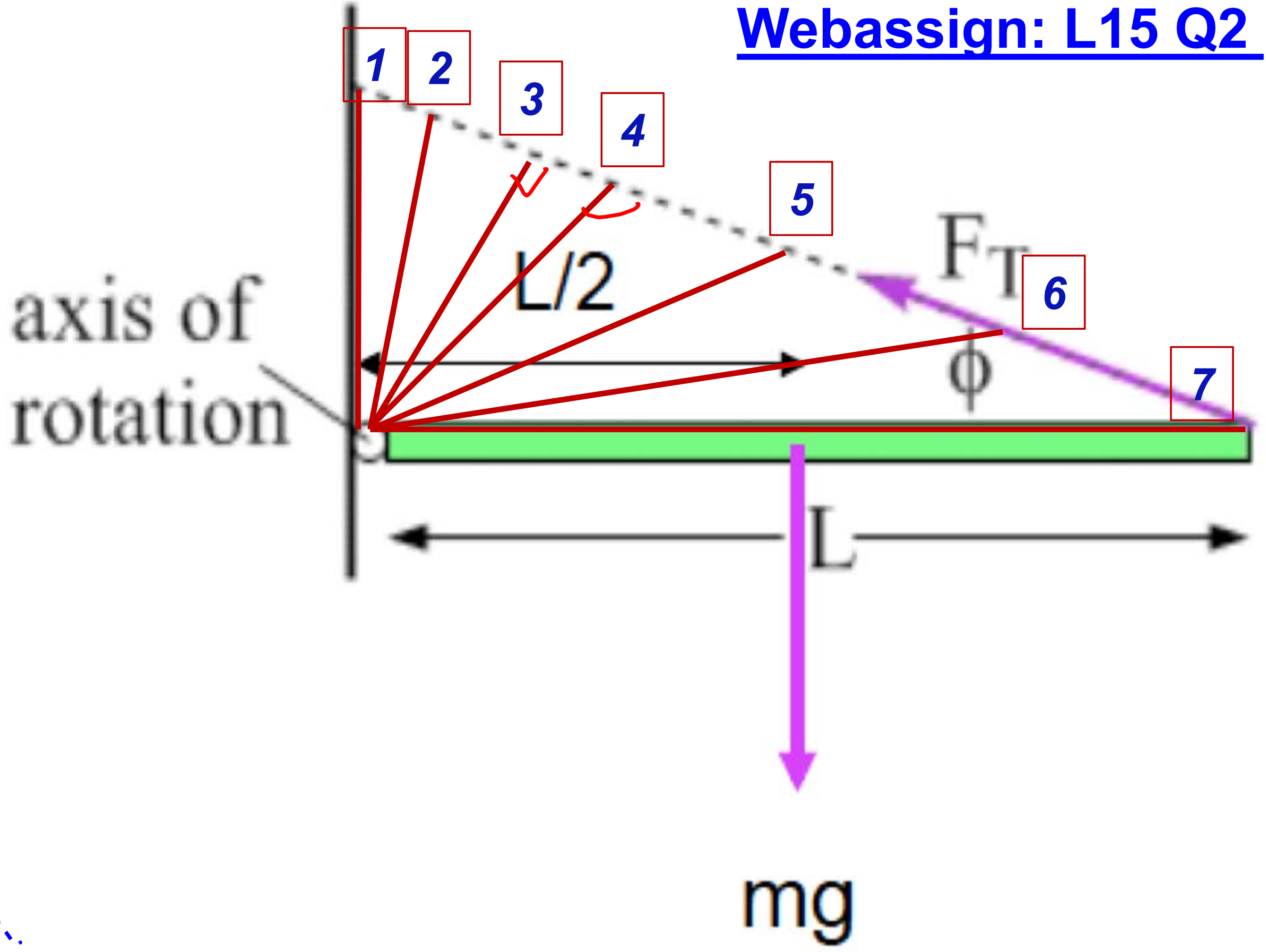
[**Webassign: L15 Q2**]

Webassign: L15 Q2

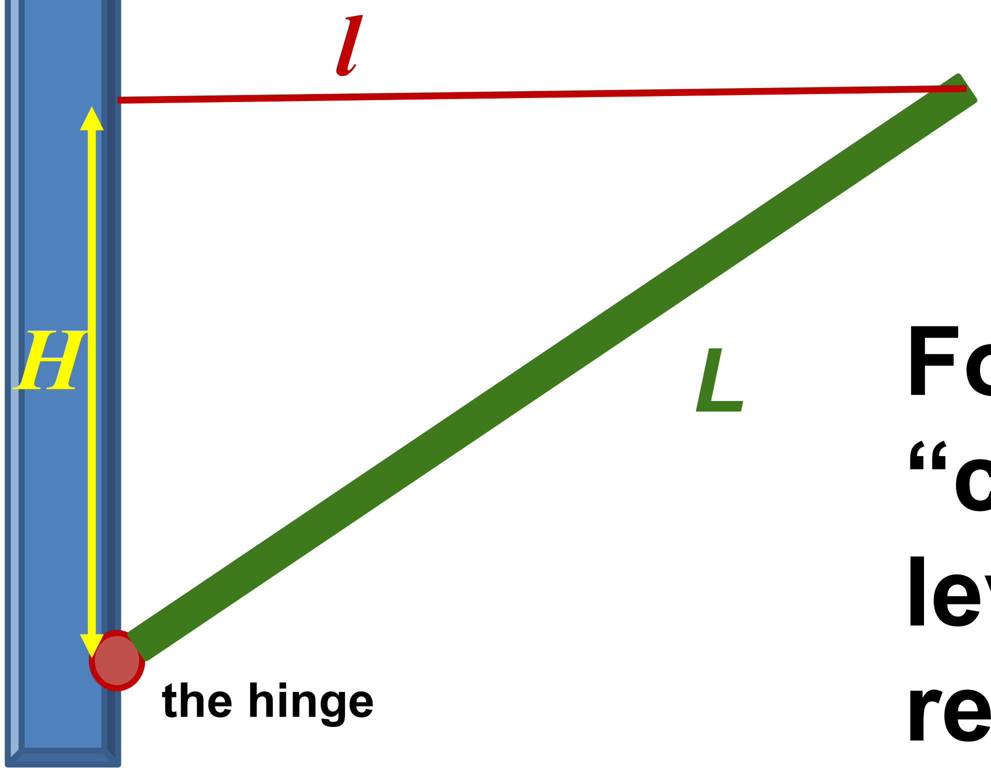
The lever-arm for the torque of the tension force relative to the hinge is represented by the red segment #

1. 2. 3. …



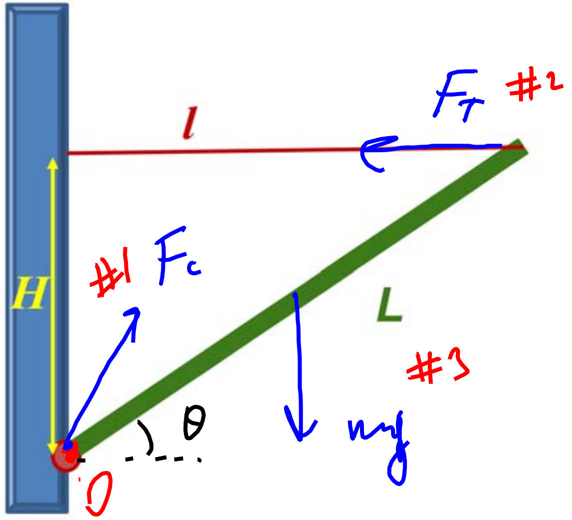


A heavy rod is suspended to a wall by the means of a horizontal string.



Draw FBD.

For each force "calculate"/draw the lever-arm relative to the hinge.



$M = 500$ gram $L = 40$ cm $\theta = 30°$

Calculate: F_T in the string; Contact Force in the hinge

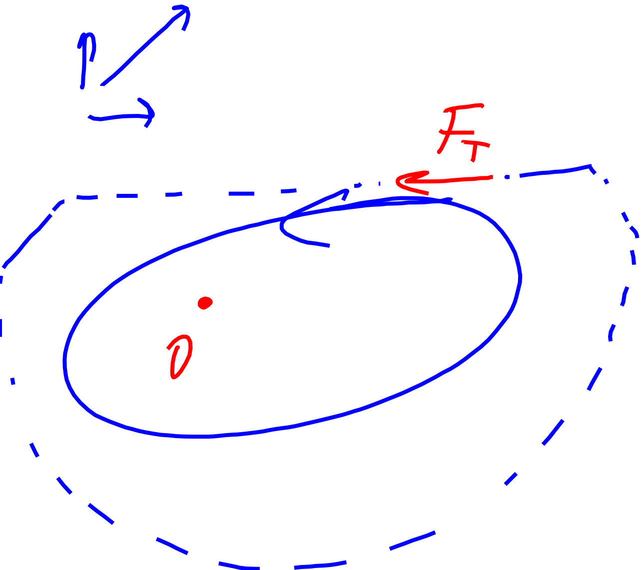
$\tau_1 = \underbrace{0 \cdot F_c}_{} = 0$

$\tau_2 = + F_T \cdot l_{F_T}$

"CCW" or "CW"

$\tau_3 = - mg \cdot l_{mg} = -5 \cdot 10 \cdot l_{mg} = -5 \cdot l_{mg}$

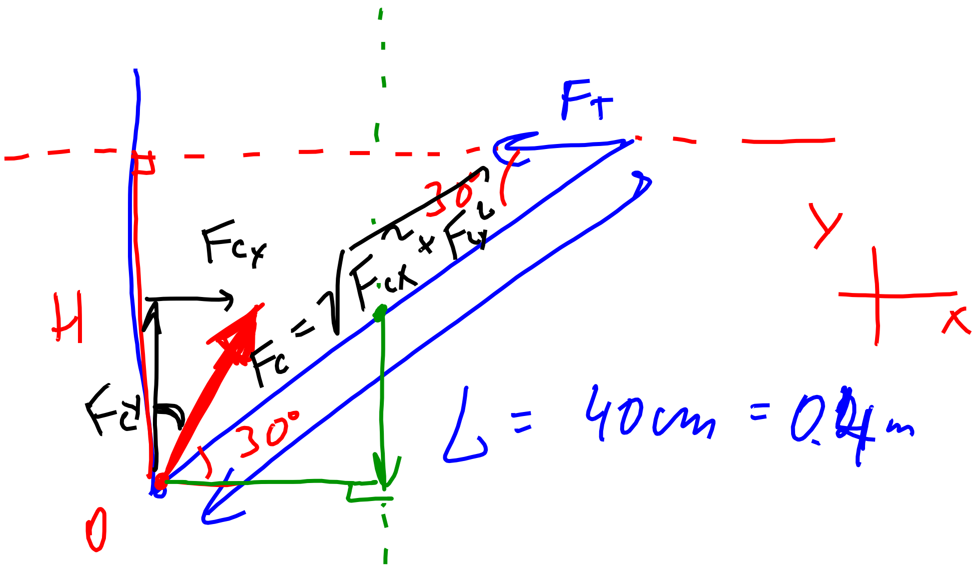
"CW"



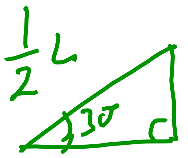
$l_{F_T} = \underline{\qquad\qquad}$; $l_{mg} = \underline{\qquad\qquad}$

TBE: $\tau_1 + \tau_2 + \tau_3 = 0 \Rightarrow 0 + F_T \cdot l_{F_T} + -5 \cdot l_{mg} = 0$

$$F_T = 5 \cdot \frac{l_{mg}}{l_{F_T}} =$$



$L = 40\,cm = 0.4\,m$



$\ell_{mg} = \frac{1}{2}L\cdot\cos 30° = 0.17\,m = 0.2\cdot\cos 30°$

$$F_T = 5\cdot\frac{\ell_{mg}}{\ell_{F_T}} = \frac{0.2\cdot\cos 30}{0.2}\cdot 5$$

$F_T = 5\cdot\cos 30°\ N$

$H = \ell_{F_T} = L\cdot\sin 30 = 0.4\cdot\frac{1}{2} = 0.2\,m$

FBE

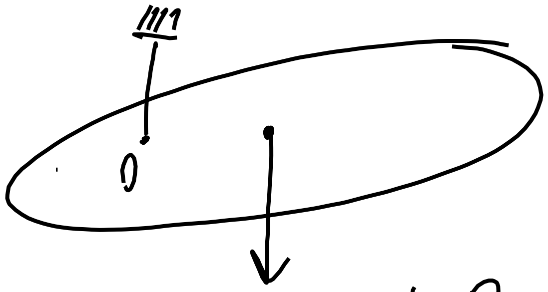
$\vec{F_c} + \downarrow mg + \overleftarrow{F_T} = \emptyset$

$\uparrow F_{cy} + mg = 0$

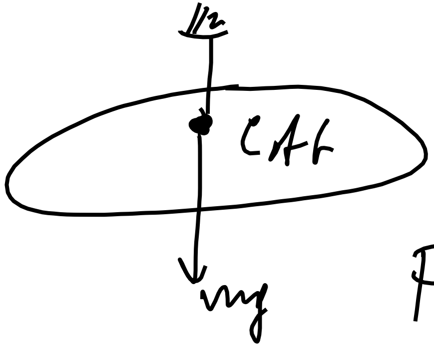
$F_{cy} = mg = 5\,N$

$x:\ \overrightarrow{F_{cx}} + \overleftarrow{F_T} = 0 ;\ F_{cx} = F_T = 5\cdot\cos 30°$

Finding the center of gravity (CG) (old name for CofM)



$\tau_{mg} \neq 0$

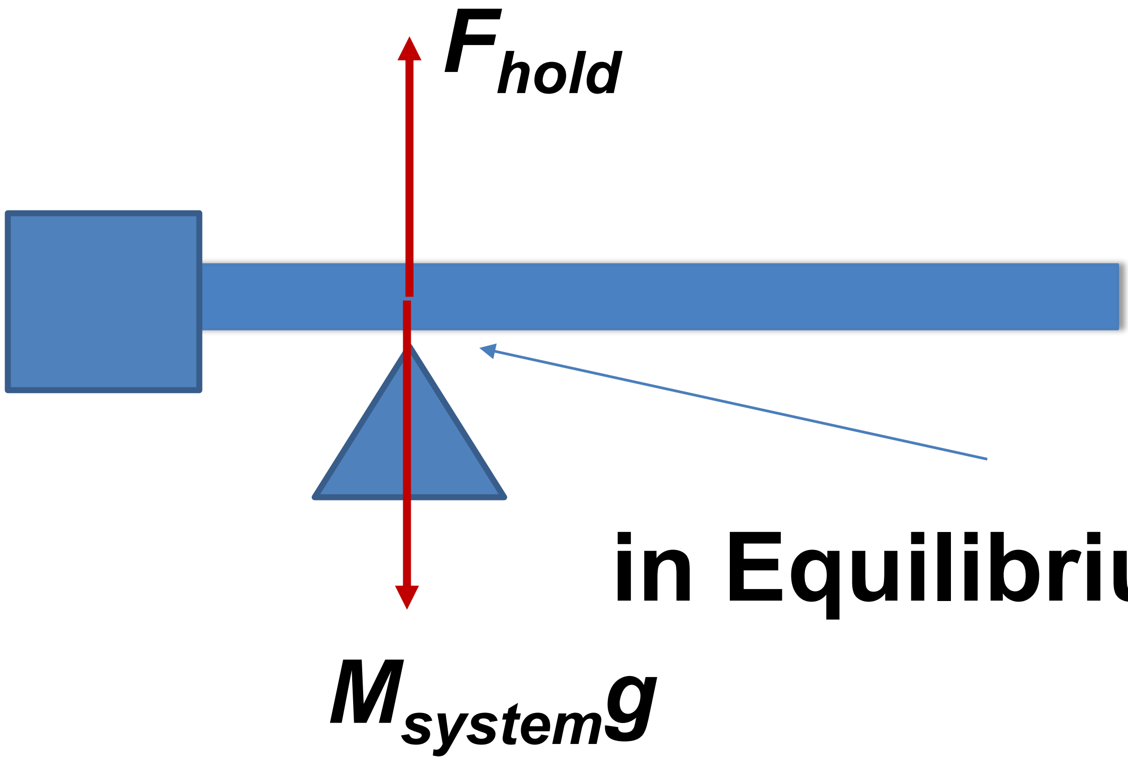


$F_{Net} = 0$

$\tau_{Net} = 0$

If this is CG the object must be in equilibrium

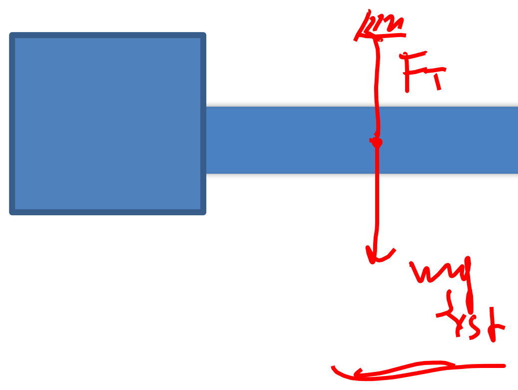
Finding the center of gravity (CG)

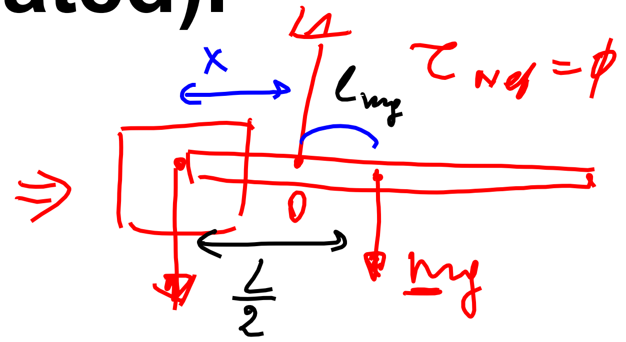


in Equilibrium => CG

Finding the center of gravity (CG)

A 100 g meter stick has a 200 g weight attached to its end. Find the CG of the system (The size of the weight is greatly exaugurated).





Mg

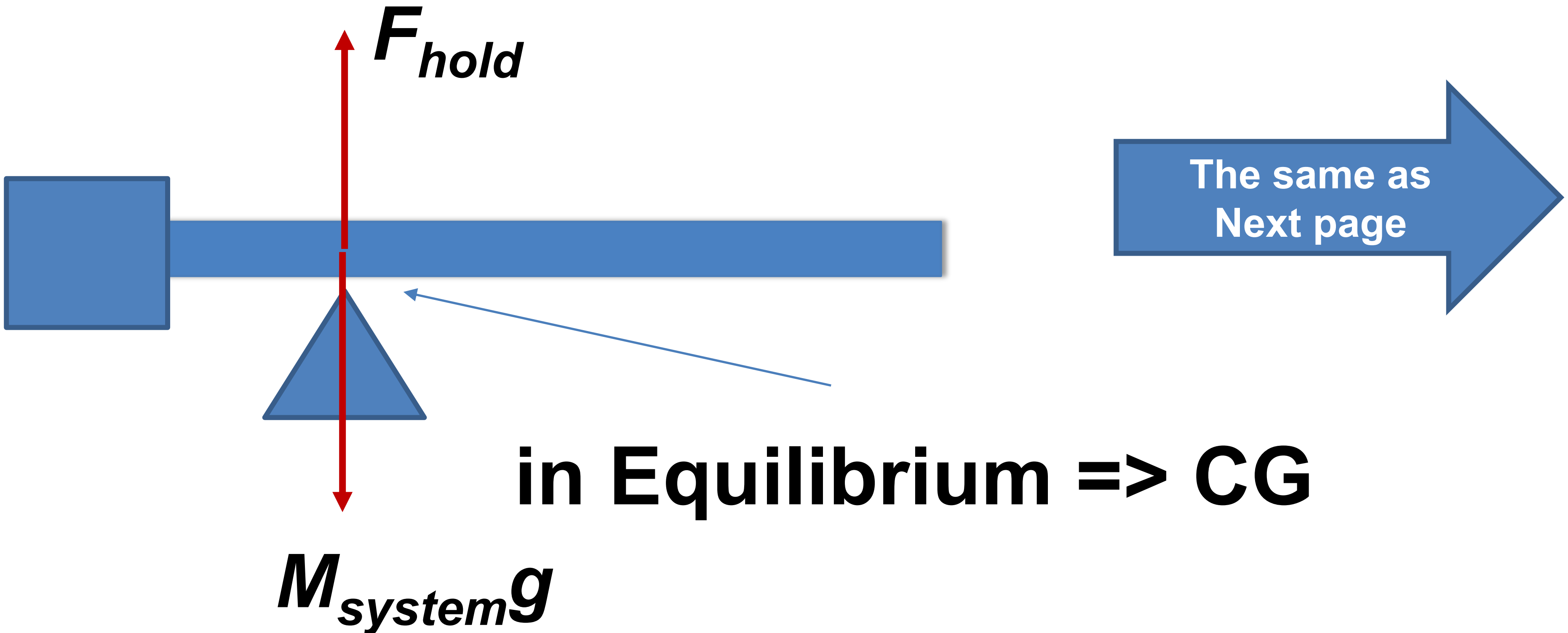
$M\cancel{g}\,x = m\cancel{g}\cdot(.5 - x)$

$2\cancel{0}\cancel{0}\cdot x = 1\cancel{0}\cancel{0}\cdot\left(\frac{1}{2} - x\right)$

$2x = \frac{1}{2} - x$; $3x = \frac{1}{2}$; $\underline{x = \frac{1}{6}}$

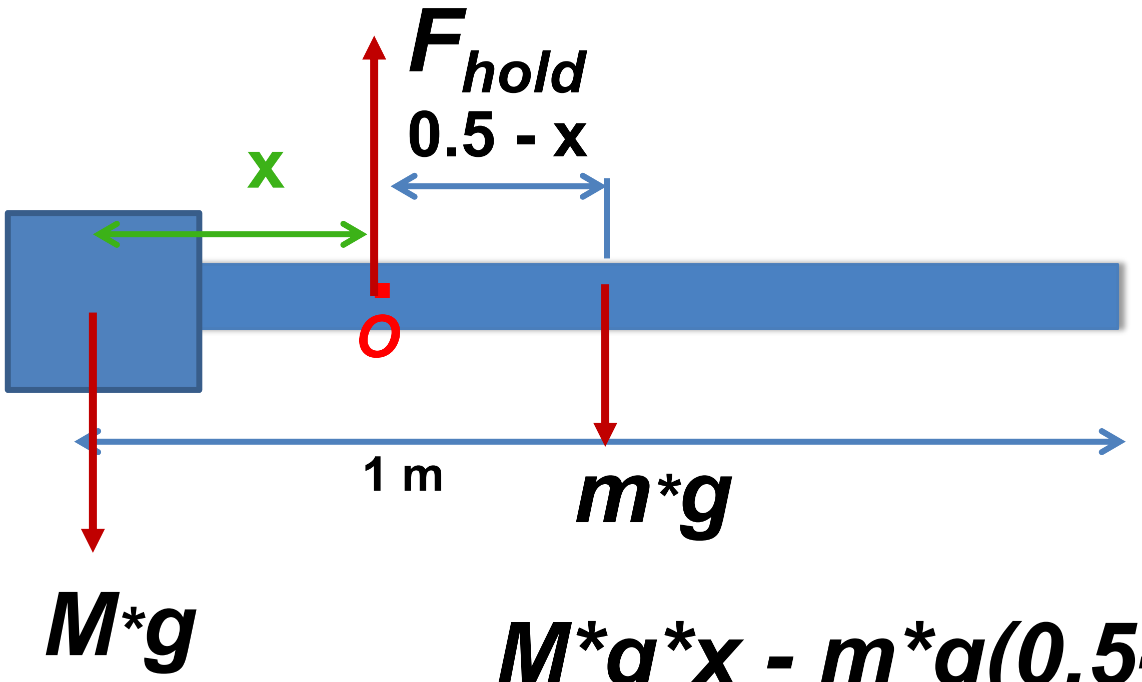
Finding the center of gravity (CG)

A 100 g meter stick has a 200 g weight attached to its end. Find the CG of the system.



Finding the center of gravity (CG)

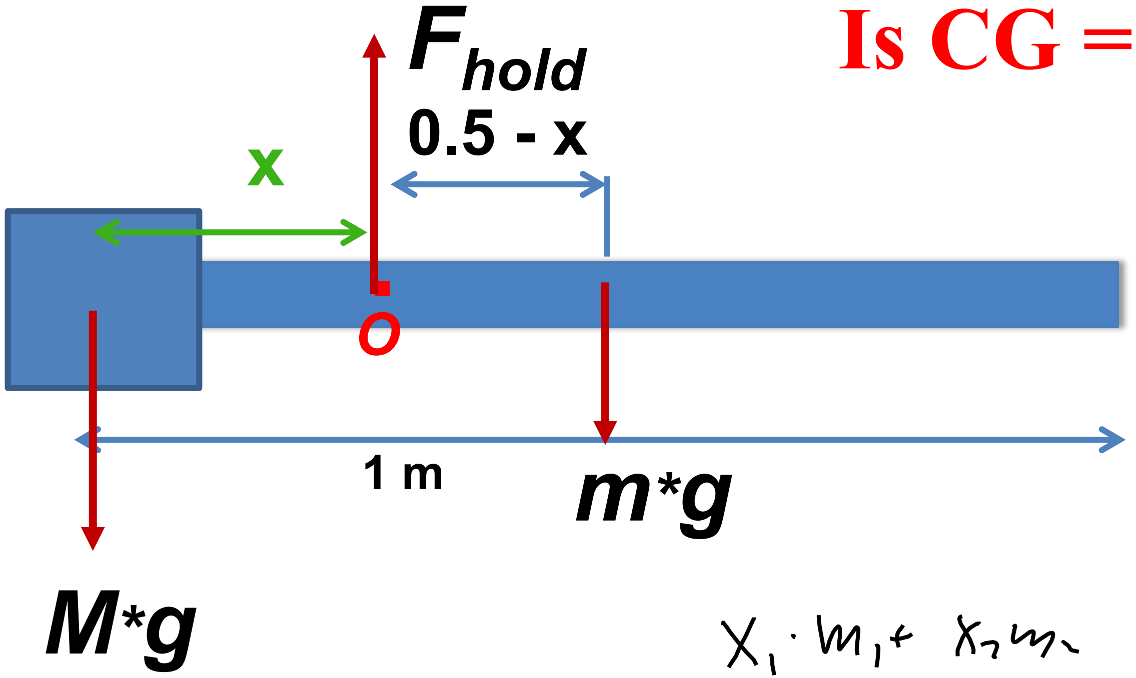
A 100 g meter stick has a 200 g weight attached to its end. Find the CG of the system.

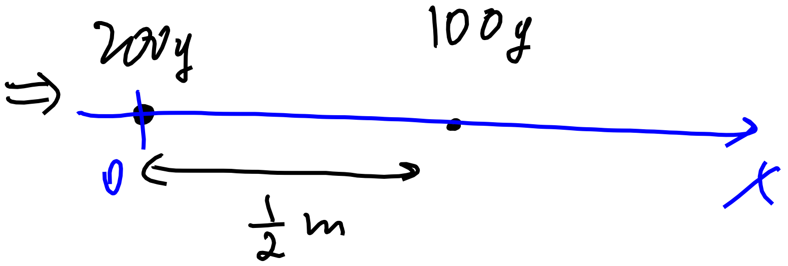


$$M*g*x - m*g(0.5-x) = 0$$

$$\Rightarrow x = 1/6\ m$$

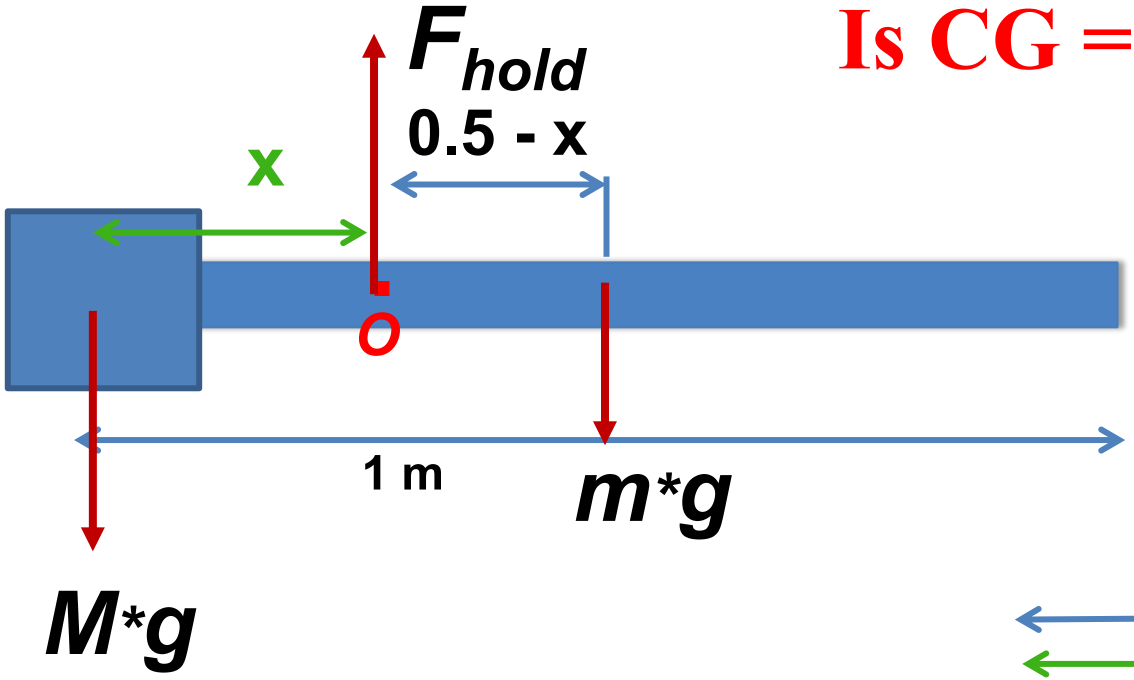
Is CG = CofM?

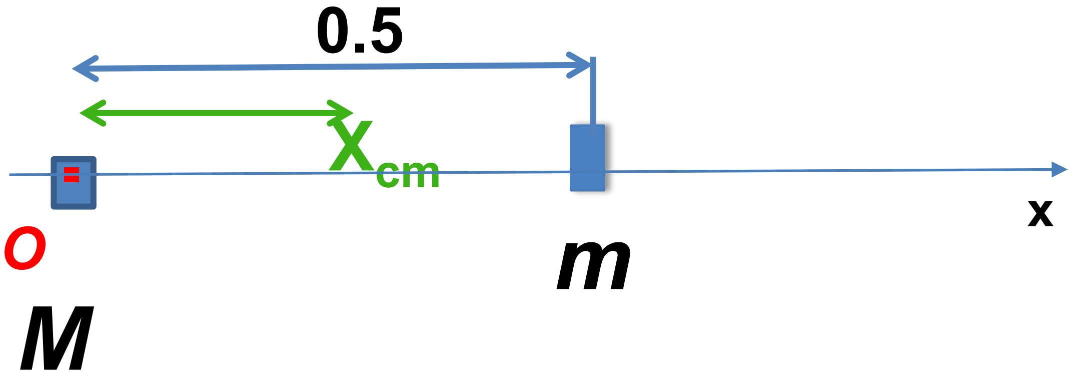




$$X_{cm} = \frac{x_1 \cdot m_1 + x_2 m_2}{m_1 + m_2} = \frac{0 \cdot 200 + \frac{1}{2} \cdot 100}{200 + 100} = \underline{\underline{\frac{1}{6}\ m}}$$

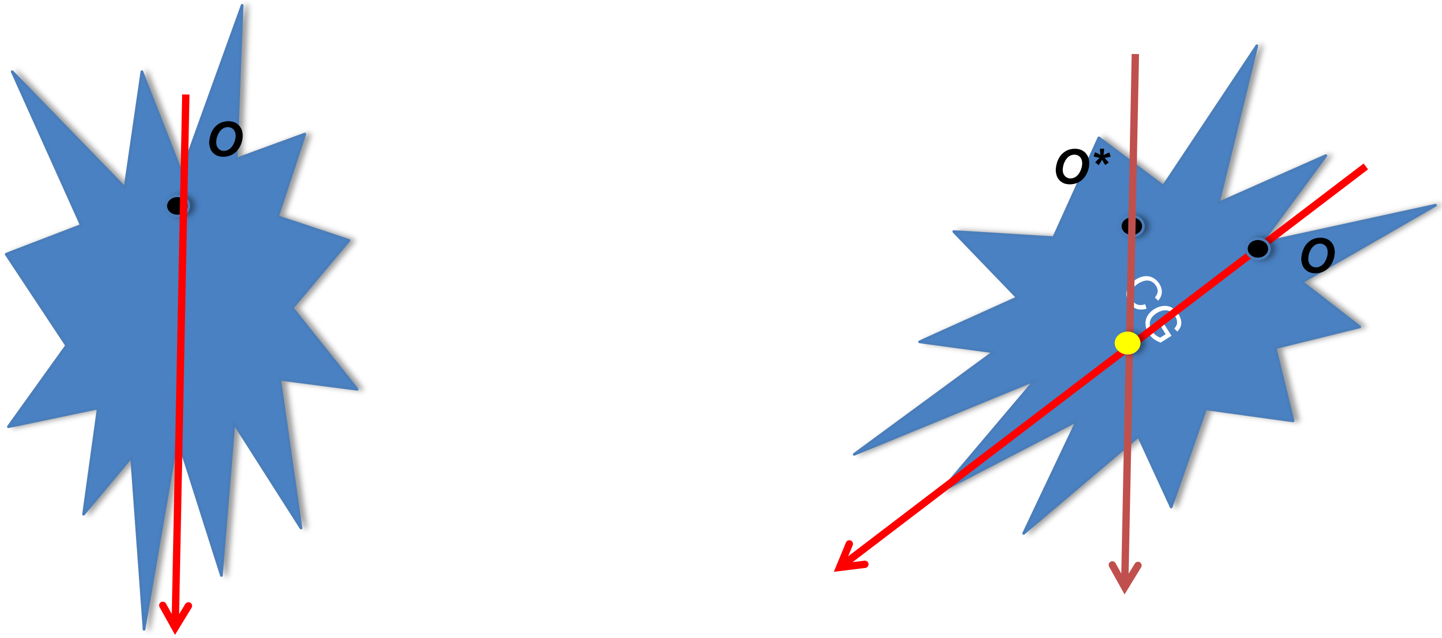
Is CG = CofM? => YES!



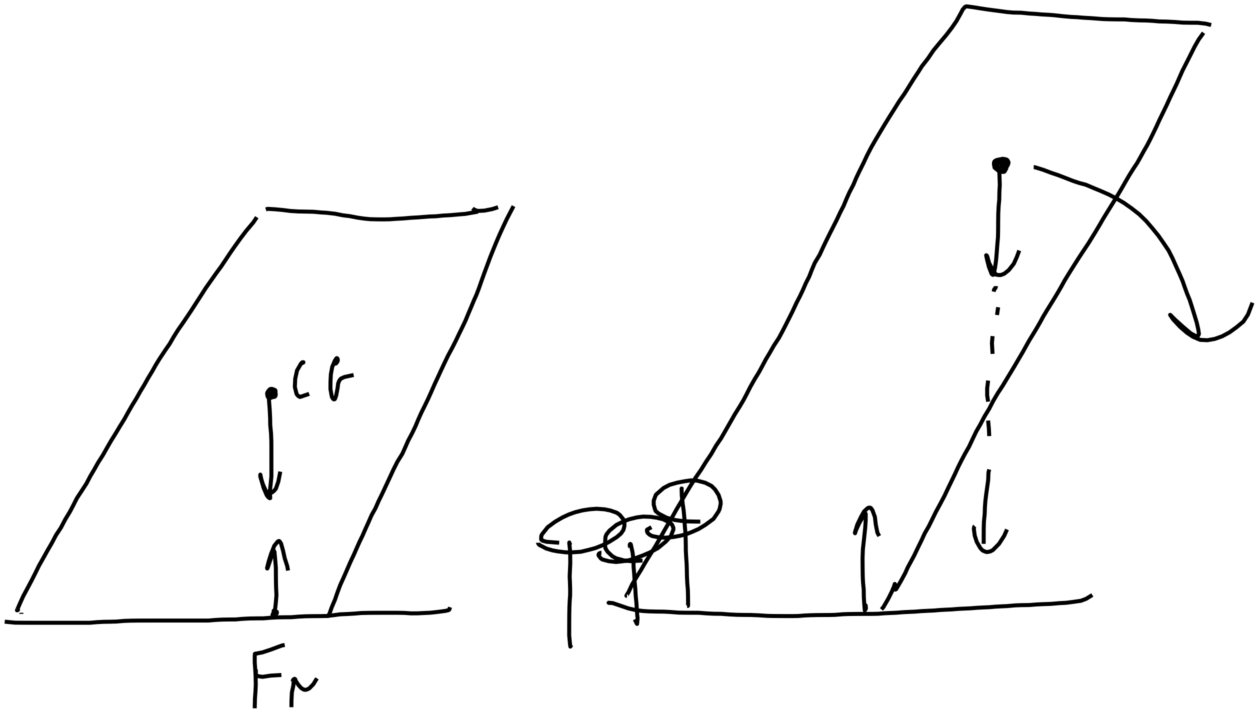


$$X_{cm} = \frac{m_1X_1 + m_2X_2}{m_1 + m_2} = 1/6 \text{ m}$$

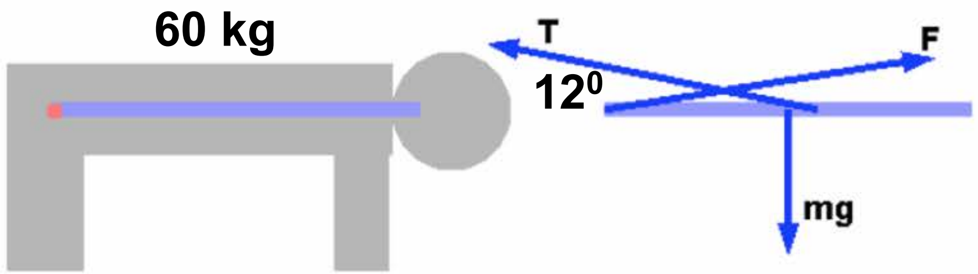
How to find the location of CG.



Why do towers fall?



https://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa



The human spine

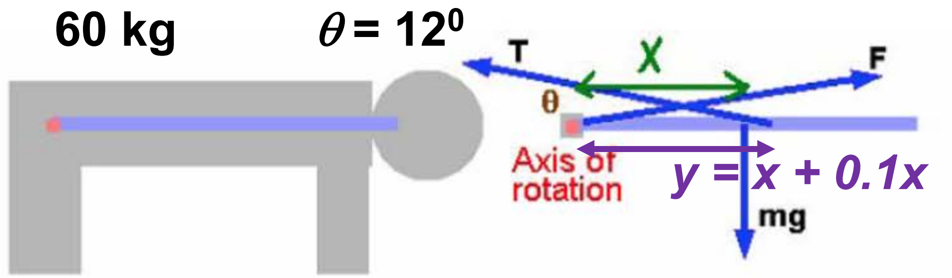
A "fun" fact

Do not read this slide

When one bends the upper body over so it is horizontal that can put a great deal of stress on the *lumbrosacral disk*, the disk separating the lowest vertebra from the tailbone (the *sacrum*). One has to be careful when picking something up!

The force of gravity, mg, acting on the upper body (this is about 65% of the body weight).
The tension in the back muscles can be considered as a force T that acts at an angle of about 12° to the horizontal when the upper body is horizontal.
The support force F from the tailbone, which also acts at a small angle measured from the horizontal.

(by A. Duffy)



At thee equilibrium

$$\Sigma\tau = 0$$

T is applied about 10% further from the tailbone than the force of gravity is, so this gives:

$$T \sin\theta * (x + 0.1x) = mg\ x$$

And $$T = \frac{mg}{1.1\sin\theta} \approx \frac{0.65 \cdot 60 \cdot 10}{1.1\sin 12} = 1706N$$

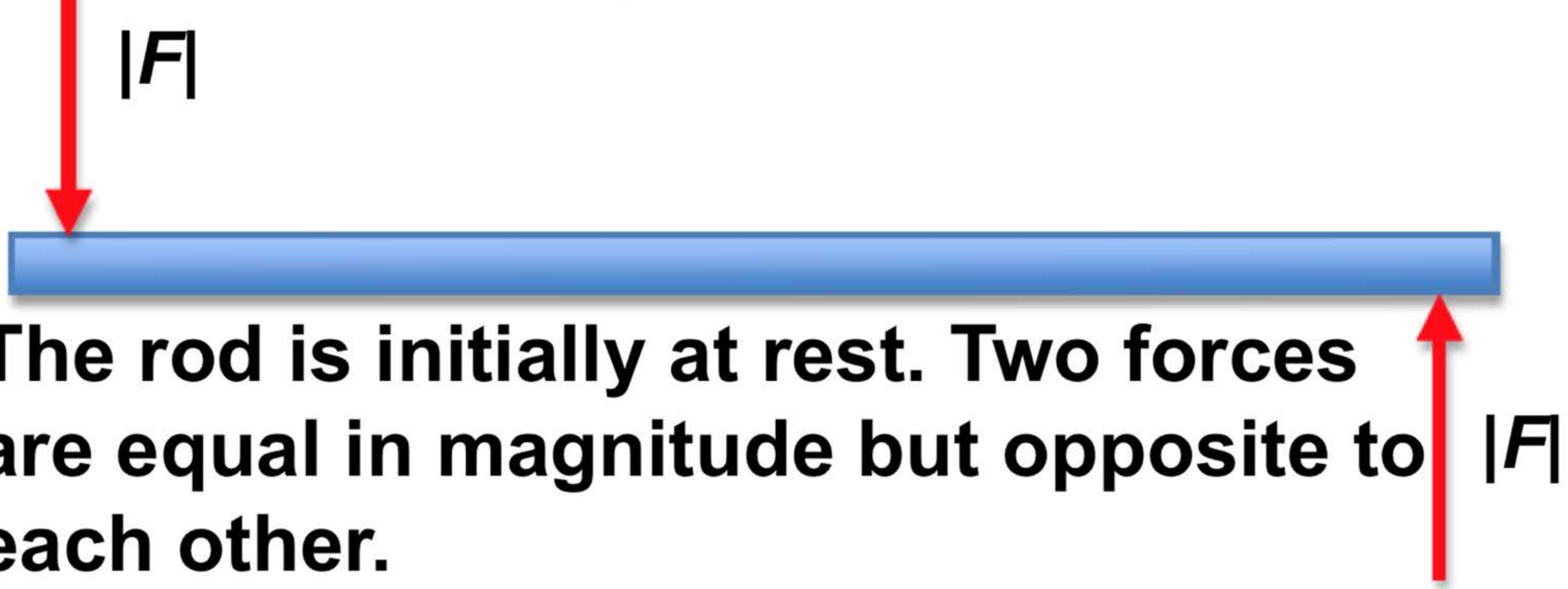
This is roughly the force pressing the one's spine!

An equivalent of about 170 kg or 370 lb!

The moral of the story: when picking up stuff, use your legs, not your back; bend your legs instead of your back

(by A. Duffy)

Not **Equilibrium**

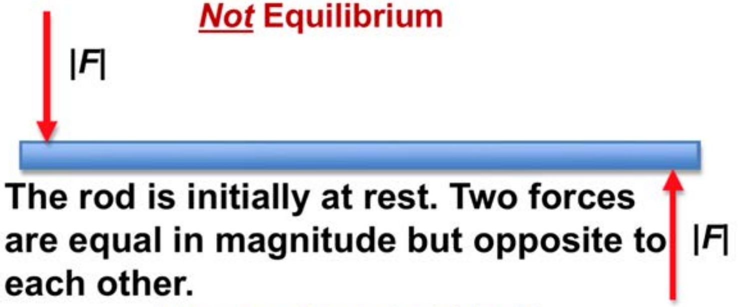


The rod is initially at rest. Two forces are equal in magnitude but opposite to each other.

The *net* *force* is ZERO!

The *net* *torque* is *NOT* zero.

=> The rod is ***rotating!*** $\alpha \neq 0$!



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The *net* ***force*** is ZERO!

The *net* ***torque*** is ***NOT*** zero.

=> The rod is ***rotating!***

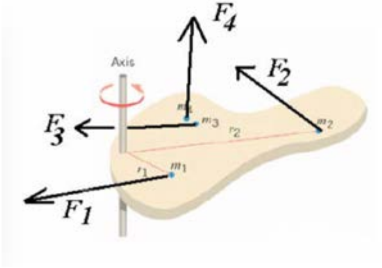
$\alpha \neq 0$!

NOT equilibrium (“pure” rotation)

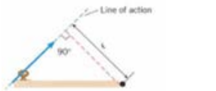
N2LR

$$\boxed{\vec{\tau}_{net} = I\vec{\alpha}}$$

$$\vec{\tau}_{NET} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots$$



CCW
Positive torque
“out” $\odot$



CW
Negative torque
“into” $\otimes$

$$I_{hollow\ cyl} = I_{ring} = mr^2$$

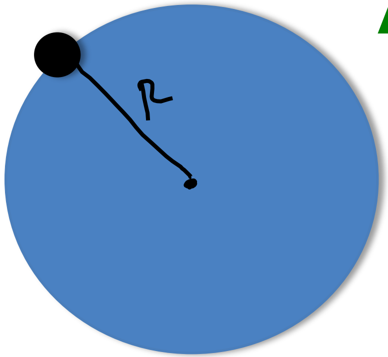
$$I_{rod} = \frac{1}{12}mL^2$$

$$I_{sph} = \frac{2}{5}mr^2$$

$$I_{disc} = I_{cyl} = \frac{1}{2}mr^2$$

I = a moment of inertia, a.k.a. rotational inertia

A disk with a ball.



$$I_{ring} = mr^2$$

$$I = \Sigma mr^2 \qquad I_{small\,mass} = mr^2 \qquad I_{sph} = \frac{2}{5}mr^2$$

$$I_{cyl} = \frac{1}{2}mr^2 \qquad I_{rod} = \frac{1}{12}mL^2$$

***R* = 20 cm, m = 50 g,**

M_{disk} = 200 g.

Find I_{system} *relative to the center of the disc.*

$$I_{sys} = I_{cyl} + I_{mass} = \frac{1}{2}M\cdot r^2 + m\cdot r^2 =$$

$$= \frac{1}{2}\cdot 2\cdot(.2)^2 + .05\cdot(.2)^2 = 0.006 \ \underline{kg\cdot m^2}$$

Webassign: L15 Q3

In order to *increase* the moment of inertia of the system we need to move the weights …

1. **Closer to the middle point of the rod**
2. **Farther away from the middle point of the rod**

$$I = \Sigma mr^2$$

$$I_{small\,mass} = mr^2$$

$$I_{sph} = \frac{2}{5}mr^2$$

$$I_{ring} = mr^2$$

$$I_{cyl} = \frac{1}{2}mr^2$$

$$I_{rod} = \frac{1}{12}mL^2$$

In order to <u>increase</u> the moment of inertia of the system we need to move the weights …

1. **Closer to the middle point of the rod**
2. **Farther from the middle point of the rod**

<u>Webassign: L15 Q3</u>

$$I_{small\,mass} = mr^2$$

$$I_{rod} = \frac{1}{12} mL^2$$

$$I_{sys} = \underline{\frac{1}{12} m \cdot L^2} + m_1 r_1^2 + m_2 r_2^2 \; ; \; r_2, r_1 \uparrow \Rightarrow I_{sys} \uparrow$$

For ANY system, in general $I = \Sigma mr^2$

If we do not change its mass, but change its *mass distribution*, we change its moment of inertia: by moving mass *away* from the axis of rotation, we *increase* the moment of inertia relative to that axis.

E.G.: a solid cylinder vs. a hollow cylinder (or a ring)

$$I_{cyl} = \frac{1}{2}mr^2$$

$$I_{ring} = mr^2$$

A sphere and a disk (or a solid cylinder) have ***the same mass and the same size***. You apply **the same net torque** to both. Starting from rest, which spins faster 1 s later?

1. Sphere
2. Cylinder
3. Equally fast

Webassign: L15 Q4

$$\vec{\tau}_{net} = I\vec{\alpha}$$

$$I_{cyl} = \frac{1}{2}mr^2$$

$$I_{sph} = \frac{2}{5}mr^2$$

A sphere and a disk (or a solid cylinder) have the same mass and the same size. You apply the same net torque to both. Starting from rest, which spins faster 1 s later?

1. Sphere
2. Cylinder
3. Equally fast

Webassign: L15 Q4

$$\vec{\tau}_{net} = I\vec{\alpha}$$

$$I_{cyl} = \frac{1}{2}mr^2 \qquad I_{sph} = \frac{2}{5}mr^2$$

$\omega \leftrightarrow \alpha$ → τ, → I

$I \uparrow \Rightarrow \downarrow \alpha$

τ = the same

$\alpha \downarrow \Rightarrow \omega \downarrow$

t = the same

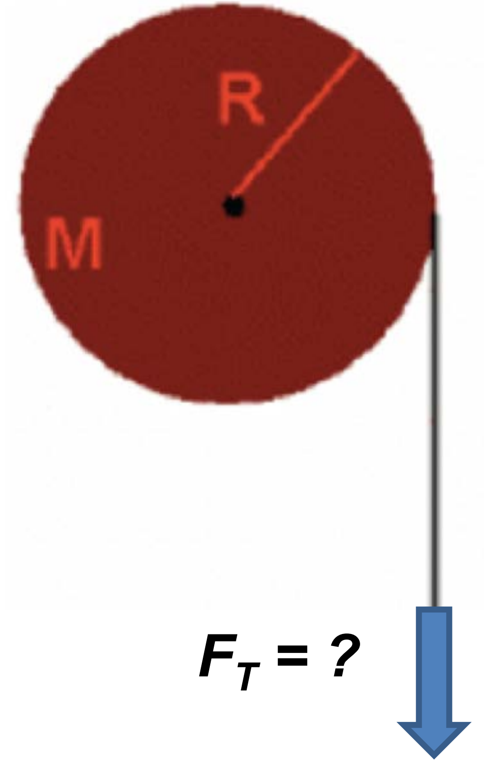
$I = C \cdot m \cdot r^2$

$I \uparrow \Rightarrow \omega \downarrow$

$(m \uparrow \Rightarrow v \downarrow)$

$\frac{1}{2} > \frac{2}{5}$

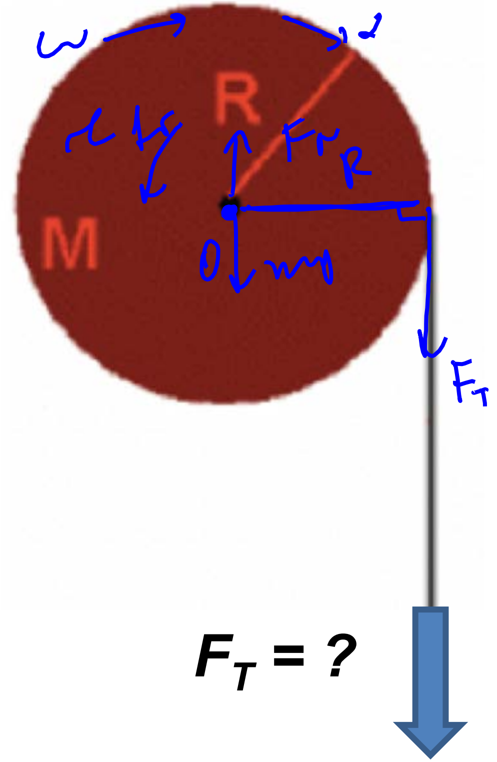
If M = 10 kg, R = 25 cm, what force should we apply to a string to provide for the disc the angular acceleration of 20 rad/s²?



$$I_{cyl} = \frac{1}{2} m r^2$$

$$\vec{\tau}_{net} = I \vec{\alpha}$$

If M = 10 kg, R = 25 cm, what force should we apply to a string to provide for the disc the angular acceleration of 20 rad/s²?



$F_T = ?$

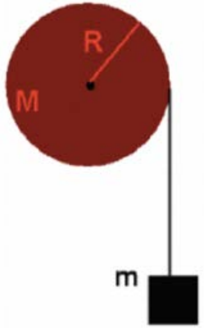
$$I_{cyl} = \frac{1}{2}mr^2$$

$$\left|\vec{\tau}_{net}\right| = \left|I\vec{\alpha}\right|$$

$\tau_{F_T} = \tau_{net}$; $F_T \cdot R = \frac{1}{2} m \cdot R^2 \cdot \alpha$

$$F_T = \frac{1}{2} \cdot \frac{mR^2\alpha}{R} = \frac{mR\alpha}{2}$$

$$F_T = \frac{10 \cdot .25 \cdot 20}{2} = \frac{200}{8} = 25\,N$$



Two N2Ls!

Newton's 2nd law for *translational* motion (CofM)

$$\vec{F}_{net} = m\vec{a}$$

Newton's 2nd law for *rotational* motion (usually about CofM)

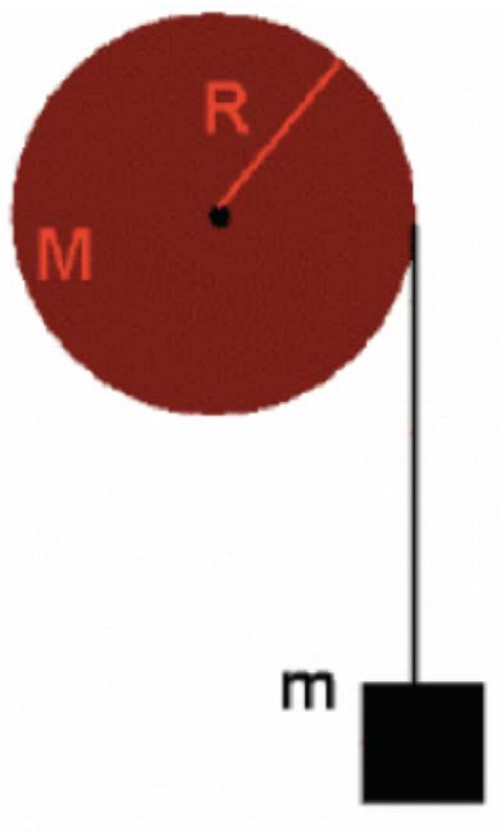
$$\vec{\tau}_{Net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots$$

$$\vec{\tau}_{net} = I\vec{\alpha}$$

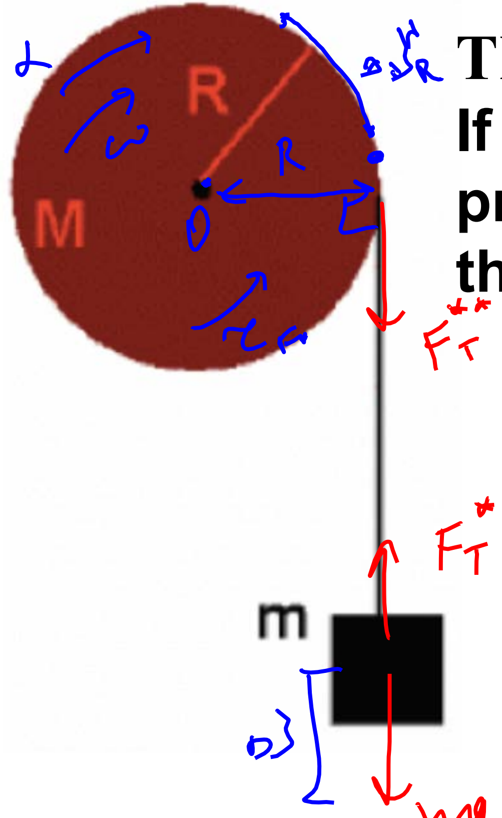
I **– moment of inertia**

Depends on the shape (mass distribution)

The disk was released from rest.



If *M* = 10 kg, *m* = 5 kg, *R* = 25 cm, assume friction provides a torque equal to 0.1 of the torque provided by the force of tension. Find the angular acceleration.



The disk was released from rest.

If M = 10 kg, m = 5 kg, R = 25 cm, assume friction provides a torque equal to 0.1 of the torque provided by the force of tension. Find the angular acceleration.

N2L: 1) $F_T - mg = m\cdot(-a)$

2) N2LR: $\tau_{fr} - \tau_{F_T} = I\cdot(-\alpha)$

3) $|\tau_{fr}| = .1 \cdot \tau_{F_T}$

4) $\tau_{F_T} = F_T \cdot R$

$\Delta s = \Delta s_R$

$\Downarrow$

5) $\alpha = \frac{a_T}{R} = \frac{a}{R}$

$mg = 5\cdot 10 = 50N$

$|F_T^{*}| = |F_T^{**}| = F_T$

Physics is done!

$F_T - m \cdot g = -ma$

$\tau_{fr} - \tau_{F_T} = I \cdot (-\alpha)$

$\alpha = \frac{a}{R}$ $\qquad \tau_{fr} = 0.1 \cdot \tau_{F_T}$

$\tau_T = F_T \cdot R$

$0.1 \cdot F_T \cdot R - F_T \cdot R = -\alpha I$

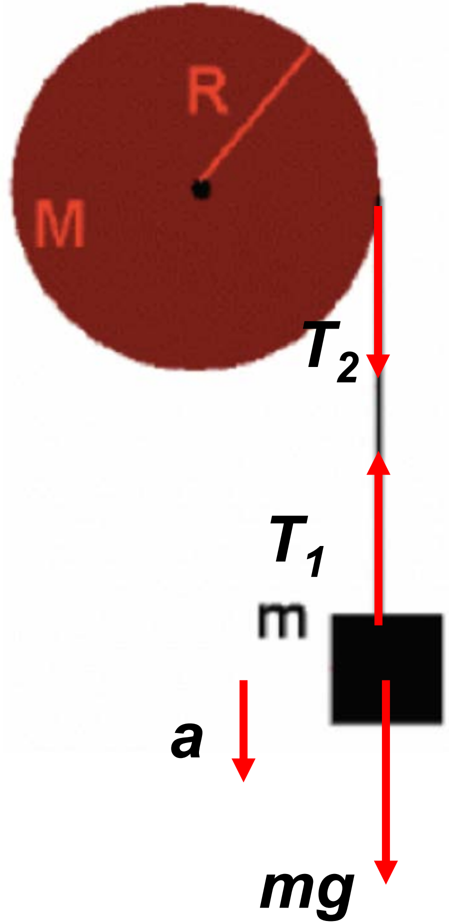
$F_T = \dfrac{-\alpha I}{0.1 \cdot R - R}$

$\dfrac{-\alpha I}{0.1 \cdot R - R} - mg = -ma$

$a = \alpha \cdot R$

$\dfrac{-\alpha \cdot I}{0.1R - R} - mg = -m\alpha R$

$a \to v$, s, t $\qquad \alpha$ $\qquad \alpha$: ω, t, θ



$|T_1| = |T_1| = T$

$$mg - T = ma$$

$$T \cdot r - 0.1 * T \cdot r = \frac{1}{2} M r^2 \cdot \frac{a}{r}$$

$$0.9 * T = \frac{1}{2} M \cdot a$$

$$mg = ma + \frac{1}{2}\frac{Ma}{0.9} = \left(m + \frac{M}{1.8}\right) a$$

$$a = \frac{mg}{m + \frac{M}{1.8}} = \frac{5 * 10}{5 + \frac{10}{1.8}} = 4.7 \frac{m}{s^2}$$

$$T = m(g-a) = 5 \cdot (10 - 4.7) = 26.5 N$$