## No labs today

Please, login into webassing, locate LectureMCQ_L18 (PY105)
and answer question 1
(but ONLY Q1!).
Pleas sign in using the sign-in sheets on the bench.

## Good morning!

Please, pick up YOUR exam,
Thank you

## LectureMCQ_L18 Question 2 !

Please, asses your expectation regarding the exam 1. The Exam was much harder than I expected
2. The Exam was somewhat harder than I expected
3. The Exam was about as I expected
4. The Exam was somewhat easier than I expected
5. The Exam was much easier than I expected
6. The Exam was way too short.
7. When did we have it? Did I miss an exam?
8. I took PY211 exam by mistake, it was OK

## Exam problems similar

## Train yourself in recognition!

Some helpful guestions for asfing phyaice problems (page e 12 1. What objects are involved? What processes are happening to them? use your imagination - make a picture showing the objects and the processes they are involved intol
. What properties of the objects and the processes might be importans?
3. What physical quantities should be used for describing those properties, what connections might be important?
5. What laws or definitions should be used to describe importan connections mathematically?
. How can I solve my equations mathematically?
Does it make a sense?
Could I solve a aimilar proble?
httpiliseschology xyz/general_algorithm.htm

## Problems: 1.HW <br> 2. Lectures 3.Units (IL)

Practice HW
Practice exams

## New topics (do not read this slide)

SHM, stable equilibrium, restoring force, oscillations, small oscillations, Hooke's law, Newton's $2^{\text {nd }}$ law for SHM, simple harmonic motion (SHM), SHM for horizontal spring, analogy between SHM and UCM, motion equation for SHM,S, V, A graphs for SHM, period, frequency, angular frequency, amplitude, elastic potential energy, energy graphs, conservation of energy, SHM for a vertical spring, a simple pendulum, SHM for a simple pendulum, a physical pendulum; fluids, density, pressure, pressure in a static fluid, atmospheric pressure, gauge pressure, absolute pressure, the Pascal's law, the buoyant force, Archimedes' principle, A static equilibrium for objects in liquid, solving buoyancy problems, fluid dynamics, an ideal fluid, streamline flow, an incompressible fluid, mass flow rate, volume flow rate, the continuity equation, the Bernoulli's equation, solving fluid dynamics problems.

HW3P1 recommended deadline $=6 / 2211 \mathrm{pm}$ actual deadline $=6 / 2811 \mathrm{pm}$

HW3P2 recommended deadline $=6 / 2411 \mathrm{pm}$ actual deadline $=6 / 2811 \mathrm{pm}$

HW3P3 recommended deadline $=6 / 26 \quad 11 \mathrm{pm}$ actual deadline $=6 / 2811 \mathrm{pm}$

HW3P4 recommended deadline $=6 / 27 \quad 11 \mathrm{pm}$ actual deadline $=6 / 2811 \mathrm{pm}$


Always choose the origin at the equilibrium position !!!!
Restoring force always points at the equilibrium position !!!!

$$
\begin{aligned}
& \text { Hooke's law; } \\
& \text { setting the origin } \\
& \text { at the equilibrium } \\
& m a_{x}=F_{x} \quad F_{X}=-k x \\
& a_{x}=-\frac{k}{m} x \quad \frac{k}{m}=\omega^{2} \quad a_{x}=-\omega^{2} x \quad \frac{a_{x}}{x}=-\omega^{2}
\end{aligned}
$$



## For an object on a spring

## Dynamics of SHM



Restoring force always points at the equilibrium position !!!!

## For ANY SHM

$$
a_{x}=-\omega^{2} x
$$

## Simple Harmonic Motion v. Rotational Motion



## SHM v. Rotational Motion



$$
x=R^{n} \cdot \cos \left(\theta_{0}+\omega t\right)
$$

## SHM v. Rotational Motion

For rotational motion $\boldsymbol{\sim} \boldsymbol{\sim} \quad$ For oscillations

$X=R \cos (\theta)=A \cos \left(\omega t+\theta_{0}\right) \hookleftarrow \quad$ "Our"motion equation

## The blast from the ...trigonometry

## $\cos \left(35^{\circ}\right)=0.819$

$$
\begin{aligned}
& -\cos \left(35^{\circ}+\underline{180}\right)=\underline{0.819} \\
& \sin \left(35^{\circ}+\underline{90}\right)=\underline{0.819}
\end{aligned}
$$

## The blast from the ...trigonometry

$$
\begin{gathered}
\cos \left(35^{\circ}\right)=0.819 \quad-\cos \left(35^{0}+180\right)=0.819 \\
\sin \left(35^{0}+90\right)=0.819
\end{gathered}
$$

$$
\cos (\theta)=-\cos \left(\theta+180^{\circ}\right)=\sin \left(\theta+90^{\circ}\right)
$$

cos, sin, -cos, -sin can be converted into each other!

## SHM v. Rotational Motion



## A general motion equation $x(t)$ for a SHM

$\boldsymbol{x}(\boldsymbol{t})=[$ number $] *\{\sin$ or $\cos \}([$ number $] * \boldsymbol{t}+[$ number $])+[$ number $]$
An order may differ due to : A + B = B + A and $A B=B A$

## examples

$$
\begin{array}{lll}
x(t)=3 \cos (2 t) & x(t)=3 \cos (5 t) & x(t)=3 \cos (-5 t) \\
x(t)=-3 \cos (2 t) & x(t)=-3 \cos (2-5 t) \quad x(t)=-3 \sin (2-5 t) \\
x(t)=3 \sin (2 t) & x(t)=-3 \cos (-5 t+2)+1.23456 \\
x(t)=-3 \sin (2 t) & x(t)=-1.23456+\left(\frac{1}{3}\right) \sin (-0.5+7.7 t)
\end{array}
$$

## Webassign: L18 Q3

## How many equations from the equations below do NOT describe SHM?

$$
\begin{array}{ll}
\text { 1. } x(t)=3 \cos (2 t) & \text { 2. } x(t)=3 \cos (5 t) \quad \text { 3. } x(t)=-3 \cos (2 t) \\
\text { 4. } x(t)=-3(2-5 t) & \text { 5. } x(t)=-3 \cos (-5 t+2)+1.23456 \\
\text { 6. } x(t)=3 \tan (2 t) & \text { 7. } x(t)=-1.23456+\left(\frac{1}{3}\right) \sin (-0.5+7.7 t)
\end{array}
$$

$x(t)=[$ number $] *\{\sin$ or cos $\}([$ number $] * t+[$ number $])+[$ number $]$

## How many equations from the equations below do NOT describe SHM?

## => 2

$$
\begin{array}{ll}
\text { 1. } x(t)=3 \cos (2 t) & \text { 2. } x(t)=3 \cos (5 t) \quad \text { 3. } x(t)=-3 \cos (2 t) \\
\text { 4. } x(t)=-3(2-5 t) & \text { 5. } x(t)=-3 \cos (-5 t+2)+1.23456 \\
\text { 6. } x(t)=3 \tan (2 t) & \text { 7. } x(t)=-1.23456+\left(\frac{1}{3}\right) \sin (-0.5+7.7 t)
\end{array}
$$

## SHM <br> Rotational Motion: <br> in general

 For rotational motion $\sim R=$ For oscillations For rotational motion $\underset{\sim}{\sim} \downarrow$ For oscillations

## SHM

$A=$ the maximum displacement from the equilibrium

$$
\omega=\text { const } \quad \theta=\theta_{0}+\omega t
$$

## MWMM

$$
X=A \cos \left(\omega t+\theta_{0}\right)+E
$$

## SHM <br> if $E=0$

$A=$ the maximum displacement from the equilibrium

$$
\omega=\text { const }
$$

$$
\theta=\theta_{0}+\omega t
$$



$$
\omega=\frac{2 \pi}{T}=2 \pi f
$$

$$
X=A \cos \left(\omega t+\theta_{0}\right)
$$

$$
x(t)=A \cos (\omega t+\theta)+E
$$

$A$ is the amplitude, which is the magnitude of the maximum
displacement from the equilibrium position.
$T=t / N=1 / f=2 \pi / \omega$ is a period, which is the time for one complete oscillation (one complete oscillation has four similar parts)
$f=N / t=1 / T$ is the frequency, i.e. the number of oscillations per one second.
$\omega=2 \pi f=2 \pi / T$ is an angular frequency, which is the number of oscillations per $2 \pi$ seconds.
$\omega t+\theta_{0}$ is a phase; $\theta_{0}$ is an initial phase (no need to bother with it)
$E$ is the coordinate of the position between $X_{\max }$ and $X_{\min }$
$x(t)=A \cos (\omega t+\theta)+E$
$A$ is the amplitude, which is the magnitude of the maximum

For an obiect on a snrina
 displacement from the equilibrium position. $T=t / N=1 / f=2 \pi / \omega$ is a period, which is the time for one complete oscillation (one complete oscillation has four similar parts)
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$\omega t+\theta_{0}$ is a phase; $\theta_{0}$ is an initial phase (no need to bother with it)
$E$ is the coordinate of the position between $X_{\max }$ and $X_{\min }$

## SHM in general: motion equation

 and a graph
$x(t)=A \cos \left(\omega t+\theta_{0}\right)+E$


SHM in general: a
$x(t)=A \cos \left(\omega t+\theta_{0}\right)+E$ motion equation and a graph
httos://www.desmos.com/calculator

$$
n=f=v=\frac{N}{t}=\frac{1}{T} \quad T=\frac{t}{N} \quad \omega=2 \pi f=\frac{2 \pi}{T}
$$



Webassign:ل18@4 The period of this motion is
$1.1 \mathrm{~s} \quad 2.2 \mathrm{~s} \quad 3.3 \mathrm{~s} \quad 4.4 \mathrm{~s} \quad .$.
$x(\mathrm{~m}) \uparrow \mathrm{A}+\mathrm{t}=0 \quad \mathrm{x}=\mathrm{b}$
Webassign: L18 Q4

## The period of this motion is

## 1.1 s 2.2 s 3.3 s ..

$t$ (s)

$=2 s \Rightarrow T=8 s$
the summary of kinematics of SHM
$x(t)=A \cos \left(\omega t+\theta_{0}\right)+E$ parts)
$A$ is the amplitude, which is the magnitude of the maximum displacement from the equilibrium position. $T=t / N=1 / f=2 \pi / \omega$ is a period, which is the time for one complete oscillation (one complete oscillation has four similar
$f=N / t=1 / T$ is the frequency, i.e. the number of oscillations per one second.
$\omega=2 \pi f=2 \pi / T$ is an angular frequency, which is the number of oscillations per $2 \pi$ seconds.
$\omega t+\theta_{0}$ is a phase; $\theta_{0}$ is an initial phase
$E$ is the coordinate of the position between $X_{\max }$ and $X_{\min }$


$x=A \cdot \cos (\omega d=a) \cdot P^{\circ}$

$x=6 \cdot 4 \frac{\pi}{i}+1=c i \theta_{0} i \theta_{0}=0$


# This graph represents the motion of a a 100 g weight attached to a spring. 

## Calculate the spring/force constant of the spring.

## Special case:

$$
x(t)=A \cos (\omega t)
$$

1. $X_{\text {equilibrium }}=0(E=0)$ 2. Released from rest
2. $X_{0}=A>0$

$$
\text { At } t=0
$$



THE MOTION DIAGRAM


## Special case:

## $x(t)=A \cos (\omega t)$



1. $X_{\text {equilibrium }}=0(E=0)$
2. Released from rest
3. $X_{0}=A>0$

$$
\text { At } t=0
$$

GRAPHS $x(t), v(t) \quad V=-V_{\text {man }} \cdot \operatorname{Sin}(\underline{\omega} t)$

## Special case:

## $x(t)=A \cos (\omega t)$



GRAPHS $x(t), a(t)$


1. $X_{\text {equilibrium }}=0(E=0)$
2. Released from rest
3. $X_{0}=A>0$

$$
\text { At } t=0
$$



Graphs of position, velocity, and acceleration
In SHM (simple harmonic motion), the general equations for position, velocity, and acceleration are:

$$
\begin{gathered}
\mathrm{x}(\mathrm{t})=\mathrm{A} \cos (\omega \mathrm{t}) \\
\mathrm{v}(\mathrm{t})=-\mathrm{A} \omega \sin (\omega \mathrm{t}) \\
\boldsymbol{a}_{\boldsymbol{x}}=-\boldsymbol{\omega}^{2} \boldsymbol{x} \quad \mathrm{a}(\mathrm{t})=-\mathrm{A} \omega^{2} \cos (\omega \mathrm{t})
\end{gathered}
$$



The phase angle $\theta_{0}$ is determined by the initial position and initial velocity.
The angular frequency for an object of mass $m$ oscillating on a spring of spring constant k the angular frequency is given by:
$\omega^{2}=\frac{\mathrm{k}}{\mathrm{m}} \quad$ The most important connection
Whatever is multiplying the sine or cosine represents the maximum valu of the quantity.
Thus: $\quad \mathrm{x}_{\max }=\mathrm{A}$
$\mathrm{v}_{\text {max }}=\mathrm{A} \omega$
$\mathrm{a}_{\max }=\mathrm{A} \omega^{2}$

$$
a_{x}=-\omega^{2} x
$$

## A special case (summary):

A cart is attached to a spring, we move a cart to the right from ( $E=0$ ) equilibrium and release it from rest.


## This graph represents the motion of a a 100 g weight attached to a spring.

## Calculate $v_{\text {max }}$ and $a_{\text {max }}$

$x(f)=A c e s(e t+a)+E$
the summary of kinematics of SHM
$A$ is the amplitude, which is the magnitude of the maximum displacement from the equilibrium position
$T=t / N=1 / f=2 \pi / \omega$ is a period, which is the lime for one complete oscillation (one complete oscillation has four similar parts)
$f=N / t=1 / T$ is the frequency, i.e. the number of oscillations per one second.
$\omega=2 \pi f=2 \pi / T$ is an angular frequency, which is the number of oscillations per $2 \pi$ seconds.
ot $+\theta_{0}$ is a phase; $\theta_{0}$ is an initial phase
$E$ is the coordinate of the position between $X_{\max }$ and $X_{\min }$

$$
\begin{aligned}
& a_{m m}=\left|\cdot \omega^{2} \cdot A\right|=A \cdot \omega^{2}= \\
= & 6 \cdot\left(\frac{\pi}{4}\right)^{2} \mathrm{~m} / \mathrm{s}^{2} \\
V_{m}= & \omega \cdot A=\frac{\pi}{4} \cdot 6 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Graphs of position, velocity, and acceleration
In SHM (simple harmonic motion), the general equations for position, velocity, and acceleration are:

$$
\begin{array}{cc}
\boldsymbol{\Delta} \boldsymbol{x}=\boldsymbol{x}-\boldsymbol{E} & \mathrm{x}(\mathrm{t})=\mathrm{A} \cos \left(\omega \mathrm{t}+\theta_{0}\right)+\boldsymbol{E} \\
\boldsymbol{F}_{\boldsymbol{x}}=-\boldsymbol{k} \boldsymbol{\Delta} \boldsymbol{x} & \mathrm{v}(\mathrm{t})=-\mathrm{A} \omega \sin \left(\omega \mathrm{t}+\theta_{\mathrm{o}}\right) \\
\boldsymbol{a}_{\boldsymbol{x}}=-\boldsymbol{\omega}^{2} \boldsymbol{\Delta} \boldsymbol{x} & \mathrm{a}(\mathrm{t})=-\mathrm{A} \omega^{2} \cos \left(\omega \mathrm{t}+\theta_{0}\right)
\end{array}
$$



In general:
A cart is attached to a spring, we move a cart away from equilibrium and release it with a push.

Whatever is multiplying the sine or cosine represents the maximum value of the quantity.
Thus: $\quad \mathrm{x}_{\text {max }}=\mathrm{A} \quad \mathrm{v}_{\max }=\mathrm{A} \omega \quad \mathrm{a}_{\max }=\mathrm{A} \omega^{2}$

The motion equation of an object is
$x(t)=-5 \sin (3 \pi t-1)+2 \quad$ (assume SI units)

## $X=A \cos \left(\omega t+\theta_{0}\right)+E$ <br> Webassign:1-18 Q5

The angular frequency equals (in rad/s)...
$1.1 \pi \quad 2.2 \pi$
3. $3 \pi$
4. $4 \pi$
5. $5 \pi$
$6.6 \pi$
$7.7 \pi$
8. none of the above

$$
\omega=\frac{2 \pi}{T}=2 \pi f
$$

## The motion equation of an object is

$$
\omega=\frac{2 \pi}{T}=2 \pi f
$$

The angular frequency equals (in rad /s)...
$1.1 \pi$ 2. $2 \pi$
4. $4 \pi$
5. $5 \pi$
$7.7 \pi \quad$ 8. none of the above
$6.6 \pi \quad 7.7 \pi$

$$
A=|-5|=5
$$

$$
a_{m-x}=w^{?} A=(3 x): 5: \quad V_{\text {max }} \text { wa }=55.5
$$

Also find: $A, f, \omega, v_{\max }, a_{\max }$

The motion equation of an object is
$x(t)=-5 \sin (3 \pi t-1)+2$

$$
\omega=3 \pi \mathrm{rad} / \mathrm{s} \quad \text { (assume SI units) }
$$

The period equals (s)...
1.1/3-2.2/3
3.3/3 $4.4 / 3$

## special cases

Released from rest:

"Kicked"
from equilibrium position:

$\underline{2}^{\mathrm{ND}}$ special case $x= \pm A \sin \omega t$


$$
a_{x}=-\omega \cdot x
$$

"Kicked" from equilibrium position: $v(t), a(t)$


## An object is released from rest

A 100 g block attached to a spring with $k=10 \mathrm{~N} / \mathrm{m}$ is moved 10 cm to the left away from the equilibrium position and released from rest. Write a motion equation and a velocity equation for the block.

A 100 g block attached to a spring with $k=10 \mathrm{~N} / \mathrm{m}$ is moved 10 cm to the left away from the equilibrium position and released from rest. Write a motion equation and a velocity equation for the block.

$$
\begin{aligned}
& \xrightarrow[-A=-10 \mathrm{~cm}]{\substack{k=0}} \\
& x=\underset{-}{+} \sin _{\cos }(\omega \cdot t) \\
& x=-\operatorname{los}(10 \cdot f)
\end{aligned}
$$




# When an object is making SHM (e.g. a ball attached to an ideal spring and 

 moving on a frictionless surface), its $K E$... 1. Conserved 2. Positive when the ball is to the right to equilibrium, but negative when the ball is to the left to the equilibrium 3. Never negative 4. Never positive 5. None of the above1. $X_{\text {equilibrium }}=0(E=0)$
2. Released from rest
3. $X_{0}=A>0$

## Dynamics of SHM

 $\xrightarrow[0]{\longrightarrow}$ Energy: What is happening to KE?
## KE=0



KE


When an object is making SHM (e.g. a ball attached to an ideal spring and moving on a frictionless surface), its KE .... 1. Conserved
2. Positive when the ball is to the right to equilibrium, but negative when the ball is to the left to the equilibrium 3. Never negative 4. Never positive 5 None of the above

## SHM



When an object is making SHM (e.g. a ball attached to an ideal spring and
moving on a frictionless surface), its $K E \ldots$ 1. Conserved 2. Positive when tho-ball is to the right to equilibrium, but negative when the-balt is to the left to the equilibrium
3. Never negative 5. None-of the-above-
4. Never positive AND changes!

Dynamics of SHM

## Why does KE change?

## Because $F_{\text {el }}$ does mechanical work!

$F_{X}=-k x$
WKET: $\quad W_{\text {net }}=W_{\text {el }}=K E_{\mathrm{f}}-K E_{\mathrm{i}}$
nuns
$W_{\text {el }}=? ? ? ?$
$W_{\text {el }}=$ Work done BY a spring $W_{\text {app }}=$ Work done ON a spring

$$
W_{\mathrm{el}}=-W_{\mathrm{app}}
$$

$F_{X}=-k x$


## Dynamics of SHM

Work; KE, Elastic Potential Energy (EPE)
If $F=$ const $=>W_{F}=|F||\Delta S| \cos \theta$
Elastic force is NOT constant! Work done by the spring!

$$
W_{e l}=\frac{k x_{i}^{2}}{2}-\frac{k x_{f}^{2}}{2}
$$

$$
P E_{e l}=\frac{k x^{2}}{2}
$$

$$
\text { WKET: } \quad W_{\text {net }}=W_{\text {el }}=K E_{\mathrm{f}}-K E_{\mathrm{i}}
$$

Graphs for kinetic and potential energy: (t)

$$
K E=\frac{m v^{2}}{2}
$$

of what?


$$
P E_{e l}=\frac{k x^{2}}{2}
$$

$$
\begin{aligned}
& M E=K E+P E= \\
& =\frac{m v}{2}, \frac{K r}{2}
\end{aligned}
$$



Graphs for kinetic and potential energy: (x)


$$
V E=E_{H}-\frac{K x^{2}}{2}:=\frac{K A^{2}}{2}-\frac{K x^{2}}{2}
$$




$$
\frac{m v^{2}}{2}+\frac{k x^{2}}{2}=M E=\mathrm{const}
$$

$M E_{\text {total }}=K E+U=$ const $=K E_{1}+U_{1}=K E_{2}+U_{2}$

$\sin ^{2}(\omega t)+\cos ^{2}(\omega t)=1 \Rightarrow U+K=$ const $=E_{\text {total }}=U_{\max }=K_{\max }$

