## Good morning!

This week, Friday is on Monday schedule!

Please, sign in, login into webassing, locate LectureMCQ_L5 (PY105) and answer question 1

## NOTE: Exam 1

 is on Monday, June 4, 8:30-10:30 am, in LSE B01Hint: arrive ~8-15
(but ONLY Q1!)

## Lab3 is in SCI 134



## Vectors

A vector is an arrow!
It has a length and a direction. To describe a vector we can:

1. Set its magnitude and the angle measured from a given direction

## OR

2. Set its components (e.g. numerical)

## Vector and its components

Components of vector $\overrightarrow{\mathbf{A}}$, are such vectors $\overrightarrow{\mathbf{A}}_{x}$ and $\overrightarrow{\mathbf{A}}_{y}$, which are (a) parallel to $x$ and $y$ axes, (b) the sum of which is equal to vector $\mathbf{A}$.

A "component" = A "coordinate"
The $x$-component ( $x$-coordinate) of a vector $A$ is the number which is:
a) equal to the magnitude of its $x$-vector component, if it points parallel to the $\boldsymbol{x}$-axis
b) equal to $(-1) \mathrm{x}$ the magnitude of its $x$-vector
 component, if it points opposite to the $x$-axis.

Components of a vector are $a_{x}=-12$, and $a_{y}=-16$. Draw the vector from the origin. The head of the vector is in ... quadrant. 1. $1^{\text {st }} \quad 2.2^{\text {nd }} \quad 3.3^{\mathrm{d}} \quad 4.4^{\text {th }}$ 5. What is a "quadrant"?
6. What is
a "vector"?
7. What is a "head"?

Components of a vector are $a_{x}=-12$, and $a_{y}=-16$. Draw the vector from the origin. The head of the vector is in ... quadrant.

1. $1^{\text {st }}$
2. $2^{\text {nd }}$
3. $3^{\mathrm{d}}$
4. $4^{\text {th }}$
5. What is a "quadrant"?
6. What is
a "vector"?
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Components of a vector are $a_{x}=-12$, and $a_{y}=-16$. Draw the vector from the origin. The head of the vector is in ... quadrant. 1. $1^{\text {st }} \quad 2.2^{\text {nd }} \quad 3.3^{\mathrm{d}} \quad 4.4^{\text {th }}$ 5. What is a "quadrant"?
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a "vector"?
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VECTOR ADDITION Draw $\vec{A}+\vec{B} \quad(=\boldsymbol{A}+\boldsymbol{B})$
(little arrows vs. bold font)
$\overrightarrow{r_{f}}=\overrightarrow{r_{i}}+\Delta \vec{r}$


Two-step rule ("tail-to-head"): 1. "MOVE"; 2. "CONNECT"

Adding two vectors algebraically
$A_{x}=A \cos \theta_{A} \quad A_{y}=A \sin \theta_{A}$
$B_{x}=B \cos \theta_{B} \quad B_{y}=B \sin \theta_{B}$.

$$
\tan \theta=\frac{R_{y}}{R_{x}}
$$

$$
R=\sqrt{R_{x}^{2}+R_{y}^{2}}
$$



To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_{x}, \mathbf{A}_{y}, \mathbf{B}_{x}$ and $\mathbf{B}_{y}$ shown in the image.

using the component method


FIND A + B


$$
A_{x}=27 \cdot 5 \cdot \cos 66=
$$

$$
R_{x}=-0.05 \sim 0
$$

(a)

$$
B_{x}=-30 \cdot \cos 68
$$

$$
\begin{aligned}
& A_{y}=27 \cdot 5 \cdot \sin 66 \\
& B_{y}=30 \cdot \sin 68 \\
& R_{y}=L_{\mid R 1=52}^{52}
\end{aligned}
$$

using the component method



FIND A + B

(a)

$$
A_{x}=27.5 \cdot \cos 66=11.2
$$

$A_{y}=27.5 * \sin 66=25.1$
$B_{y}=30 * \sin 68=27.8$
$R_{x}=A_{x}+B_{x}=-0.04$

$$
R_{y}=A_{y}+B_{y}=52.9
$$

$$
B_{x}=-30 * \cos 68=-11.24
$$

$$
R=\sqrt{R_{x}^{2}+R_{y}^{2}}=53
$$

 means

$$
\begin{gathered}
R_{x}=A_{x}+B_{x}+C_{x} \\
\text { AND }
\end{gathered}
$$

$$
R_{y}=A_{y}+B_{y}+C_{y}
$$

$$
R=R_{x}+R_{y}
$$

AND


Opposite vector; subtraction.

The negative/opposite of a vector is just another vector of the same magnitude but pointing in the opposite direction. So - $\boldsymbol{B}$ is the negative of $\boldsymbol{B}$; it has the same length but opposite direction.

$$
\text { Subtraction: } \quad \overrightarrow{A-} \vec{B}=: \vec{A}+(-\vec{B})
$$

- 

$\vec{B}+(-\vec{B})=0$

## в

## LectureMCQ L5 Q3



## A-B points

## mostly...



2

$A-B=A+(-B)$

(a)

using the component method

(a)

$$
\begin{aligned}
& A_{x}=27.5 * \cos 66=11.2 \\
& B_{x}=-30 * \cos 68=-11.24
\end{aligned}
$$

$$
R_{x}=11.2--11.24=23 \quad R_{y}=25.1-27.8=-3
$$

using the component method

(a)

$$
\begin{aligned}
& A_{x}=27.5 * \cos 66=11.2 \\
& B_{x}=-30 * \cos 68=-11.24 \\
& R_{x}=A_{x}-B_{x}=22.44
\end{aligned}
$$

$$
A_{y}=27.5 * \sin 66=25.1
$$

$$
B_{y}=30 * \sin 68=27.8
$$

$$
R_{y}=A_{y}-B_{y}=-2.7
$$

$$
R=\sqrt{R_{x}^{2}+R_{y}^{2}}=23
$$

$$
\theta=\tan ^{-1}(3 / 22)=7.6^{0}
$$

## Projectile motion (PM)

## Projectile motion, properties of projectile

 motion, the range, the maximum height, the time of the fight.

## Which of the paths in the picture best represents the path of the cannon ball?



## Which of the paths in the picture best represents the path of the cannon ball?



## Which of the paths in the picture best represents the path of the cannon ball?



A bowling ball accidently falls out of the cargo bay of an airliner as it flies along in a horizontal direction. As seen from the

ground, which path would the ball most closely follow after falling the airplane?



## A race

Two balls are launched simultaneously from the same height. Ball A is eleased from rest, and drops straight down. Ball B is given an initial horizontal velocity.

Which ball hits the ground first?

1. This is too early for the morning class.
2. Ball A
3. Ball B
4. Both balls hit the ground at the same time
5. It depends on the mass of the balls.


## Hard to see?? Listen!

## A race <br> LectureMCQ L5 Q5

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## Follow your Intuition!

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5. It depends on the mass of the balls


Both balls hit the ground at the same time The horizontal component of velocity does not affect its vertical component!

This shows the motions of two identical balls-one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

## You Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land?



## Ballistic cart

A ballistic cart is a cart that shoots a ball vertically upward.

With the cart rolling at constant speed when it shoots the ball, where will the ball land? LectureMCQ L5 Q6

1. It depends on the speed of the cart.
2. In the cart
3. Behind the cart
4. Ahead of the cart
5. Impossible to tell

## Ballistic cart

A ballistic cart is a cart that shoots a ball vertically upward.

With the cart rolling at constant speed when it shoots the ball, where will the ball land?

1. It depends on the speed of the cart. $v$ ?
2. In the cart.
3. Behind the cart.
4. Ahead of the cart.
5. Impossible to tell.


The horizontal component of velocity does not affect its vertical component!

## You Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land -
(1) benind you, (2) ahead ofyou, or (3) in the barrel of the rifle

Experiments show that the vertical and horizontal motions are independent. But - WHY?

## We know that $a_{y}=-g$

"The $x$ - component of vector $a$ " is ...


$$
\begin{aligned}
& \text { LectureMCQ L5 Q7 } \\
& \text { 1. } a_{x}>0 \\
& \text { 2. } a_{x}=0 \\
& \text { 3. } a_{x}<0
\end{aligned}
$$

## "The $x$ - component of

 vector $a$ " is ...
## 1. $a_{x}>0 \quad$ 2. $a_{x}=0$



## LectureMCQ L5 Q7



$$
\begin{aligned}
& \vec{a}=\vec{a}_{x}+\vec{a}_{y} \\
& \vec{a}=\vec{a}_{x}+\vec{a}^{\psi} \\
& \vec{a}-\vec{a}=\vec{a}_{x} \\
& \theta=\vec{a}_{x}
\end{aligned}
$$

## We know that $a_{y}=-g$

"The $x$ - component of vector $a$ " is ...

$$
\text { y } \left\lvert\, \begin{aligned}
& \text { 1. } a_{\mathrm{x}}>0 \\
& \text { 2. } a_{\mathrm{x}}=0 \\
& \vec{a} \begin{array}{l}
\text { 3. } \\
\text { 3. } a_{\mathrm{x}}<0
\end{array}
\end{aligned}\right.
$$

A vertical vector has NO horizontal component


## 2-D motion

At the Earth's surface, $\bar{q}$, the acceleration due to gravity, equals $9.8 \mathrm{~m} / \mathrm{s}^{2}$ and is directed down.

$$
g=\left|a_{y}\right|=9.8 \mathrm{~m} / \mathrm{s}^{2} \sim 10 \mathrm{~m} / \mathrm{s}^{2} \Rightarrow \quad a_{y}=-g
$$

$$
v_{y}=v_{0 y}-g t \quad y \text {-axis points UP! }
$$

$a_{x}=0 \leadsto v_{x}=v_{o x}=$ constant


2-D motion

At the Earth's surface, $\bar{q}$, the acceleration due to gravity, equals $9.8 \mathrm{~m} / \mathrm{s}^{2}$ and is directed down.

$$
g=\left|a_{y}\right|=9.8 \mathrm{~m} / \mathrm{s}^{2} \sim 10 \mathrm{~m} / \mathrm{s}^{2} \Rightarrow>\quad \boldsymbol{a}_{y}=-g
$$

$$
a_{x}=0 \Longrightarrow v_{x}=v_{o x}=\text { constant }
$$

Add an $x$ or y subscript to our usual equations of 1-D motion (appropriate for constant acceleration...).

$$
\begin{aligned}
& \text { MV } \quad v_{0 x}=v_{0} \cos \theta_{v_{0}} v_{0}^{v_{0}-} \quad \text { yMCA } \\
& v_{x}=v_{0 x}+a / x t \stackrel{v_{0 y}=v_{y} \sin \theta}{\mathrm{c}_{\mathrm{onst}}}{ }^{v_{0}} v_{y}=v_{0 y}+a_{y} t \\
& x=x_{0}+v_{0 x} t+\frac{1}{2} a_{x} t^{2} \\
& y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \\
& v_{y}^{2}=v_{0 y}^{2}+2 a_{y}\left(y-y_{0}\right) \\
& \downarrow a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2} \\
& a_{x}=0
\end{aligned}
$$

IMPORTANT: When solving problems always keep the x -component data separate from the y -component data. The only thing that can be used in both sets of equations is time.
When solving problems on projectile motion; for the natural choice of the x - and y (up) - coordinates we have this:

$\theta=$ launch angle

## $v_{0}=$ the initial speed



IMPORTANT: When solving problems always keep the $x$-component data separate from the y-component data. The only thing that can be used in both sets of equations is time.
When solving problems on projectile motion; for the natural choice of the $x$ - and $y$ (up) - coordinates we have this:

| The $x$-component | The $x$-component |
| :--- | :--- |
| equations | equations |
| $v_{x}=v_{o x}$ | $v_{y}=v_{o y}-g t$ |
| $x=v_{o x} t$ | $y=h+v_{o y} t-1 / 2 g t^{2}$ |

In general, $h \neq 0$ !

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{ox}}=\mathrm{v}_{\mathrm{o}} \cos \theta \mathrm{v}_{v_{0 y}} \xrightarrow[v_{0}]{v_{0}}- \\
& \mathrm{v}_{\mathrm{oy}}=\mathrm{v}_{\mathrm{y}} \sin \theta \xrightarrow[v_{0 x}]{ }
\end{aligned}
$$

## 2-D motion = " [1-D motion] * 2 "



2D: Motion: projectile motion


## 2D: Motion: projectile motion



2D Motion: Math description

$$
\begin{gathered}
\vec{v}_{\text {ave }}=\frac{\Delta \vec{r}}{\Delta t}=\left(v_{\text {ave }}, v_{\text {ave y }}\right)=\left(\frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}\right) \\
\vec{v}_{\text {inst }}=\left(v_{\text {inst } x}, v_{\text {inst } y}\right) \\
\vec{r}=(x, y)
\end{gathered}
$$

2D Motion: property of $\bar{V}_{\text {inst }}$


## 2D Motion: property of $\vec{V}_{\text {inst }}$



## 2D Motion: property of $\vec{V}_{\text {inst }}$



Instantaneous velocity is tangent (tangential) to the trajectory! ANY trajectory!
$\boldsymbol{y} \uparrow$ At the highest point of the trajectory, the $y$-component of velocity, $v_{y}, \ldots$

$\begin{array}{lll}\text { 1. Negative } 2.0 & \text { 3. Positive }\end{array}$



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\author{

1. Negative 2.0 3. Positive
}


$$
v_{y_{-} \text {top }}=0
$$

## Learned!

# Projectile motion <br> => Practice 

The Height of a Kickoff
A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is $22 \mathrm{~m} / \mathrm{s}$. Ignoring air resistance, analyze various points in the motion.


The Height of a Kickoff case: $\mathrm{h}=0$
A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is $22 \mathrm{~m} / \mathrm{s}$. Ignoring air resistance, analyze various points in the motion.

$$
\begin{aligned}
& V_{x}=22 \cdot \cos 40 \\
& V_{y_{0}}=22 \cdot \sin 40
\end{aligned}
$$

$$
\begin{array}{ll}
x_{i} & x=X_{0}^{+0}+v_{x} \cdot t \quad \quad \quad \underline{0}=-g \\
y: & v_{y}=v_{y_{0}}-g \cdot t \\
& y=y_{0}+v_{y_{0}} \cdot t+\frac{1}{2}(-g) t^{2}
\end{array}
$$



The Height of a Kickoff
A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is $22 \mathrm{~m} / \mathrm{s}$. Ignoring air resistance, analyze various points in the motion.

$$
\begin{aligned}
& v_{o x}=v_{o} \cos \theta=(22 \mathrm{~m} / \mathrm{s}) \cos \left(40^{\circ}\right)=16.85 \mathrm{~m} / \mathrm{s} \\
& v_{o y}=v_{y} \sin \theta=(22 \mathrm{~m} / \mathrm{s}) \sin \left(40^{\circ}\right)=14.14 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$


$v_{\mathrm{ox}}=\mathrm{v}_{\mathrm{o}} \cos \theta=(22 \mathrm{~m} / \mathrm{s}) \cos \left(40^{\circ}\right)=16.85 \mathrm{~m} / \mathrm{s}$


$$
x=x_{0}+v_{x} t=0+17 . t
$$

$$
\begin{aligned}
y_{f}=0 ; y & =y_{0}+v_{y_{0}} \cdot t-\frac{1}{2} g t^{2} \\
& \psi \\
0 & =0+14 \cdot t_{t 1}-\frac{1}{2} \cdot 10 \cdot t^{2}
\end{aligned}
$$

$$
\left.0=14 . t_{\text {tor }}-5 . f_{\text {ct }}^{2}\right] \div t_{t_{t}}
$$

$$
0=14-50 \alpha_{d \alpha}
$$

$$
t_{2 d}=\frac{14}{5}=2.85
$$

$$
\begin{aligned}
& t_{u_{p}}=t_{d_{\text {on }}} ; f_{t_{n}}=2 t_{u_{p}}=2 t_{d_{\text {om }}} \\
& R=17 \cdot 2.8= \\
& f_{\text {op }}=f_{d_{d_{0}}}=\frac{1}{2} d_{2 l}=1.4 \mathrm{~s} \\
& \begin{array}{l}
y=M \cdot t-5 \cdot t^{2} \\
1
\end{array} \quad \uparrow \quad \uparrow \quad H_{\max }=14 \cdot 1.4-5 \cdot 1.4^{2} \div \ldots \\
& \begin{array}{ccc}
1 & \uparrow \\
H_{\text {mad }} & t_{\text {up }}
\end{array}
\end{aligned}
$$



$$
\begin{aligned}
& \\
& t_{\text {tel }}=2 \cdot t_{u p}=2.8 \mathrm{~s}
\end{aligned} \Rightarrow t_{\text {toc }}=t_{t_{p}}=1.4 \mathrm{~s}
$$

Projectile Motion
Find time to top.
$\overrightarrow{v_{\text {top }}}=\overrightarrow{v_{0 x}}$


| $y$ | $a_{y}$ | $v_{y}$ | $v_{o y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $-9.80 \mathrm{~m} / \mathrm{s}^{2}$ | 0 | $14 \mathrm{~m} / \mathrm{s}$ | $?$ |

$$
\begin{aligned}
& 0=v_{y}=v_{\mathrm{oy}}+\mathrm{a}_{\mathrm{y}} \mathrm{t}=(14 \mathrm{~m} / \mathrm{s})+\left(-9.8 \mathrm{~m} / \mathrm{s}^{2}\right) \mathrm{t} \\
& \left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) \mathrm{t}=(14 \mathrm{~m} / \mathrm{s}) \quad H=y_{\max }=v_{0 y} t_{u p}+\frac{-9.8 t_{u p}^{2}}{2}=10 \mathrm{~m} \\
& \mathrm{t}=(14 \mathrm{~m} / \mathrm{s}) /\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=1.428 \mathrm{~s}=1.43 \mathrm{~s} \text { time to top }
\end{aligned}
$$



$$
\begin{gathered}
y=v_{o y} t+\frac{1}{2} a_{y} t^{2} \\
0=(14 \mathrm{~m} / \mathrm{s}) t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}
\end{gathered}
$$

"Cancel" t

$$
0=2(14 \mathrm{~m} / \mathrm{s})+\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t
$$

$$
t=2.9 \mathrm{~S} \quad=\text { twice the time to top! }
$$

## Example The Range of a Kickoff

Calculate the range R of the projectile.

guy throwing a ball on the roof of a 6 m high barn. The ball is 2 m above the ground when leaving the guy's hands, and its horizontal velocity (the $x$ - component of the initial velocity) is $10 \mathrm{~m} / \mathrm{s}$.
The ball travels 20 m in horizontal direction before it hits the roof.
Try to find the following (in any order):
$\mathrm{t}=$ ? the time the ball was in the air
$\mathrm{v}_{\mathrm{iy}}=$ ? the vertical component on the initial velocity of the ball
$\mathrm{v}_{\mathrm{f}}=$ ? the final speed of the ball (the speed of the ball when it just start touching the roof)
how much higher was the ball at the highest point of its parabolic trajectory relative to the roof?
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$\mathrm{v}_{\mathrm{f}}=$ ? the final speed of the ball (the speed of the ball when it just start touching the roof)
how much higher was the ball at the highest point of its parabolic trajectory relative to the roof?

In the picture to the right you see a guy throwing a ball on the roof of a 6 m high barn. The ball is 2 m above the ground when leaving the guy's hands, and its horizontal velocity (the $x$ - component of the initial velocity) is $10 \mathrm{~m} / \mathrm{s}$.
The ball travels 20 m in horizontal direction before it hits the roof.
Try to find the following (in any order):


## Find everything!

$t=$ ? the time the ball was in the air
$\mathrm{v}_{\mathrm{iy}}=$ ? the vertical component on the initial velocity of the ball
$\mathrm{v}_{\mathrm{f}}=$ ? the final speed of the ball (the speed of the ball when it just start touching the roof)
how much higher was the ball at the highest point of its parabolic trajectory relative to the roof?


$$
\begin{aligned}
& \begin{array}{l}
\prod_{\text {"w }} \frac{x=v_{x} \cdot t \Rightarrow}{20=10 \cdot t_{\text {totel }} \Rightarrow t_{\text {Lute }}=\frac{20}{10}=25} \\
y=y_{i}+v_{y_{y_{0}}} \cdot t+\frac{1}{2}\left(-(0) \cdot t^{2}\right.
\end{array} \\
& \downarrow \\
& 6=2+V_{y_{0}} \cdot 2+\frac{1}{2}(-10) 2^{2} \\
& 4 V_{r o}
\end{aligned}
$$

In the picture to the right you see a guy throwing a ball on the roof of a 6 m high barn.

The ball is 2 m above the ground when leaving the guy's hands, and its horizontal velocity (the $x$ component of the initial velocity) is $10 \mathrm{~m} / \mathrm{s}$.

The ball travels 20 m in horizontal direction before it hits the roof.

Try to find the following (in any order):
$t=$ ? the time the ball was in the air
$\mathrm{R}=\mathrm{x}=20=10^{*} \mathrm{~T} \Rightarrow \mathrm{~T}=2 \mathrm{~s}$
$\mathrm{v}_{\mathrm{i} y}=$ ? the vertical component of the initial velocity
 of the ball

$$
Y_{f}=Y(2)=4=v_{y 0} * 2-5^{*} 2^{2} \quad \Rightarrow \quad v_{y 0}=12 \mathrm{~m} / \mathrm{s}
$$

$\mathrm{v}_{\mathrm{f}}=$ ? the final speed of the ball (the speed of the ball when it just start touching the roof) $\quad \mathrm{v}_{\mathrm{yf}}=12-10^{*} 2=-8 \mathrm{~m} / \mathrm{s}$

$$
\mathrm{v}_{\mathrm{f}}=\sqrt{\mathrm{v}_{\mathrm{x}}^{2}+\mathrm{v}_{\mathrm{yf}}^{2}}=\sqrt{10^{2}+(-8)^{2}}=12.8 \mathrm{~m} / \mathrm{s}
$$

How much higher was the ball at the highest point of its parabolic trajectory relative to the roof?

$$
\mathrm{Y}_{\max }=12 * \mathrm{t}_{\uparrow}-5\left(\mathrm{t}_{\uparrow}\right)^{2} \quad 0=12-10^{*} \mathrm{t}_{\uparrow} \quad \Rightarrow \quad \mathrm{t}_{\uparrow}=1.2 \mathrm{~s} \quad \Rightarrow \mathrm{Y}_{\max }=12 * 1.2-5(1.2)^{2}=7.2 \mathrm{~m}
$$

(in general $\quad Y_{\max }=\frac{\mathrm{v}_{\mathrm{y} 0}^{2}}{2 \mathrm{o}}=\frac{g t_{\uparrow}^{2}}{2}$ )

$$
?=7.2-4=3.2 \mathrm{~m}
$$

## Relative motion, velocity addition, "crossing a river".



Relative motion, velocity addition, "crossing a river".

A passenger moves
 relative to an airplane.
 of the passenger relative to the plane is represented by an arrow toward the rear of the plane.


$$
\overrightarrow{\boldsymbol{r}}_{31}=\overrightarrow{\boldsymbol{r}}_{32}+\vec{r}_{21}
$$

A passenger moves 4 m west relative to the airplane. If over the same time the plane moved 300 m east relative to the ground, what is the displacement of the passenger relative to the ground?

The passenger

## Relative displacement




Specific equation

1
The ground

$$
\overrightarrow{\boldsymbol{r}}_{31}=\overrightarrow{\boldsymbol{r}}_{32}+\vec{\Delta}_{21}
$$

General equation

For ANY 3 objects

## Relative velocity

General equation


## Relative velocity

The law of relative velocities
(LRV)

## For ANY three objects

$\vec{v}_{31}=\vec{v}_{32}+\vec{v}_{21}$

## LEARNED! => Practice!

