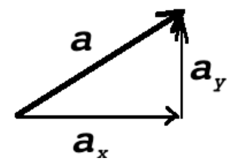
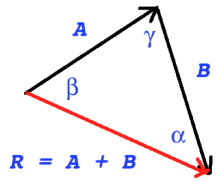
PY105 Equation Sheet II

A vector and its components:

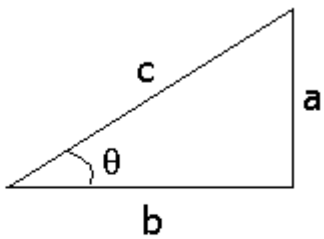


For ANY triangle:



$$R^2 = A^2 + B^2 - 2|AB|\cos\gamma \qquad \frac{|A|}{\sin\alpha} = \frac{|B|}{\sin\beta} = \frac{|R|}{\sin\gamma}$$

For a right angle triangle:



$$a^2 + b^2 = c^2 \qquad c = \sqrt[2]{a^2+b^2}$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$\sin\theta = \frac{a}{c} \qquad \cos\theta = \frac{b}{c} \qquad \tan\theta = \frac{a}{b}$$

Adding vectors: $\vec{a} + \vec{b} = \vec{c}$ => $a_x + b_x = c_x$ and $a_y + b_y = c_y$

Quadratic equation: If $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Circumference: C = 2πr Area of a circle: A = πr² Area of a triangle: A = 0.5*hb*

Conversion factors:

1 m = 100 cm 1 km = 1000 m 1 mi = 1.6 km = 1600 m 1 L = 10^{-3} m³

1 cm = 10 mm 1 min = 60 s 1 h = 60 min 1 kg = 1000 g

1 inch = 2.54 cm 1 ft = 12 inch $360^0 = 2\pi$ rad = 1 rev

General Definitions

Average Speed: Average Velocity:

$v_{avsp} = \dfrac{L}{\Delta t}$ **(*L* - distance)** $\vec{v}_{avvel} = \dfrac{\Delta\vec{r}}{\Delta t}$ ($\Delta\vec{r}$ - displacement); **(1-D)** Δx = Area[$v(t)$]

Average Acceleration: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ **(1-D)** *"Instantaneous" = "slope"* Δv = Area[$a(t)$]

Constant Acceleration Equations for 1-D Motion

$x = x_o + v_{ox}t + ½a_x t^2$ $v = v_{ox} + a_x t$ $v_x^2 = v_{ox}^2 + 2a_x(x - x_o)$ $v_{ave} = (v_0 + v_f)/2$

For the free fall (y-axis is *UP*; *g* ≈ 10 m/s²):

$v_y = v_{yi} - gt$ $y_f = y_i + v_i t - \frac{1}{2}10t^2$ $v_y^2 = v_{oy}^2 - 2g(y - y_o)$ $v_{y\,ave} = (v_{y\,0} + v_{y\,f})/2$

For the projectile motion (y - axis is UP; $g \approx 10$ m/s^2): $v_x = v_{x0} = const$

$x = x_0 + v_x t$ $v_y = v_{iy} - gt$ $y = y_o + v_{oy} t - ½ g t^2$ $v_i = \sqrt[2]{v_{ix}^2 + v_{iy}^2}$

$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$

Newton's Laws

$\vec{F}_{Net}$ **definition:** $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \ldots$ **Translational Equilibrium:** $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \ldots = 0$ **and** $\boldsymbol{v} = 0$

Newton's Second Law: $\boldsymbol{F}_{Net} = \Sigma \boldsymbol{F} = m\boldsymbol{a}$ Newton's Third Law: $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$

Friction: *kinetic* $F_{k\,fr} = \mu_k * N$ *static* $F_{s\,fr} \le \mu_s * N$ $F_{s\,fr\ max} = \mu_s * N$

Weight: $\boldsymbol{W} = m\boldsymbol{g}$ Apparent weight: $|AW| = |N|$ *or* $|AW| = |T|$

Energy and Work:

Work (constant force): $W = |F||S| \cos\theta$ (θ is the angle between force and displacement)

Kinetic Energy: $K = K_{tr} = ½ m v^2$

Work done by friction: $W = - F_{fr} L$ (L is the distance traveled)

Power: $P = \dfrac{W}{t} = |F||v| \cos\theta$

Gravitational Potential Energy (y-axis is *UP*): $U_G = mgy$ Elastic Potential energy: $U_E = ½ kx^2$

Energy Conservation:

Master equation: $U_i + K_i + W_{nc} = U_f + K_f$ where W_{nc} is work done by non-conservative forces

Work – Kinetic Energy theorem: $K_f - K_i = W_{total} = W_{net} = W_1 + W_2 + \ldots$

Systems of Particles + Momentum:

Center of mass: $X_{CoM} = \dfrac{x_1 m_1 + x_2 m_2 + \ldots}{m_1 + m_2 + ..}$; (similar expression is for Y_{CoM})

Momentum: $\boldsymbol{p} = m\boldsymbol{v}$ Impulse: $\boldsymbol{J} = \Delta\boldsymbol{p} = \boldsymbol{F}_{ave} \Delta t = $ Area $\{F(t)\}$

Linear Momentum conservation ($\boldsymbol{F}_{\text{Net System}} = 0$ or $\Delta t \approx 0$): $\vec{p}_{system\ initial} = \vec{p}_{system\ final}$

$\vec{p}_{system\ initial} = \vec{p}_1 + \vec{p}_2 + \ldots$

Elastic collision (KE = const, all pomenta point "to the right"): $v_1 + u_1 = u_2 + v_2$

Rotation:

$\theta = \dfrac{s}{r}$ $\omega = \dfrac{v}{r}$ $\alpha = \dfrac{a_t}{r}$ $N = \dfrac{S}{2\pi r} = \dfrac{\theta_{(in\,rad)}}{2\pi} = \dfrac{\theta_{(in\deg)}}{360}$

$T = t/N = 1/n$ $n = f = 1/T$ $\omega T = 2\pi$

Centripetal acceleration: $a_c = \frac{v^2}{R}$; total acceleration: $a_{total} = \sqrt{a_c^2 + a_t^2}$

Constant angular acceleration equations: $\omega = \omega_o + \alpha t$ $\theta = \theta_o + \omega_o t + ½ \alpha t^2$

Torque : $|\tau| = |rF\sin\theta| = |F| l$ (l – the shortest distance from the axis of rotation to the line of action of the force)

Equilibrium : $\sum \vec{F} = 0$ ***and*** $\sum \vec{\tau} = 0$ CCW = “positive” CW = “negative

Rotational dynamics: $\vec{\tau}_{net} = I\vec{\alpha}$ $I_{disk} = \frac{1}{2}MR^2$ $I_{sphere} = \frac{2}{5}MR^2$ $I_{rod} = \frac{1}{12}ML^2$ $I_{ring} = MR^2$

Rotational Kinetic Energy: $K_{rot} = ½I\omega^2$

Angular momentum: $L = I\omega$ (a solid object)

Angular Momentum conservation: $\vec{L}_{system\ initial} = \vec{L}_{system\ final}$

Rolling (no slipping, no non-conservative forces): $\omega = \frac{v_{CofM}}{r}$ $\alpha = \frac{a_{CofM}}{r}$

$U_i + K_i = U_f + K_f$ $K_{total} = K_{tr} + K_{rot}$

$K_{rot} = ½I\omega^2$ $K_{tr} = ½mv^2$