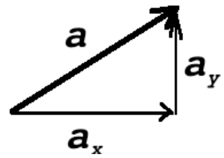
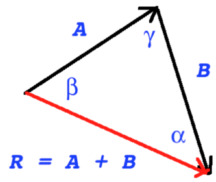
# PY105 Equation Sheet III

A vector and its components:

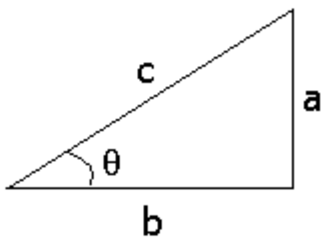



For ANY triangle:




$$R^2 = A^2 + B^2 - 2|AB|\cos\gamma \qquad \frac{|A|}{\sin\alpha} = \frac{|B|}{\sin\beta} = \frac{|R|}{\sin\gamma}$$

For a right angle triangle:




$$a^2 + b^2 = c^2 \qquad c = \sqrt[2]{a^2+b^2}$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$\sin\theta = \frac{a}{c} \qquad \cos\theta = \frac{b}{c} \qquad \tan\theta = \frac{a}{b}$$

Adding vectors: $\vec{a} + \vec{b} = \vec{c}$ => $a_x + b_x = c_x$ and $a_y + b_y = c_y$

Quadratic equation: If $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Circumference: C = 2πr** **Area of a circle: A = πr²** **Area of a triangle: A = 0.5*hb***

$\pi \approx 3$

**Conversion factors:**

1 m = 100 cm 1 km = 1000 m 1 mi = 1.6 km = 1600 m 1 L = $10^{-3}$ $m^3$

1 cm = 10 mm 1 min = 60 s 1 h = 60 min 1 kg = 1000 g

1 inch = 2.54 cm 1 ft = 12 inch $360^0 = 2\pi$ rad = 1 rev

## General Definitions

Average Speed: Average Velocity:

$v_{av\,sp} = \dfrac{L}{\Delta t}$ **(*L* - distance)** $\vec{v}_{av\,vel} = \dfrac{\Delta\vec{r}}{\Delta t}$ **(**$\Delta\vec{r}$ - displacement**);** **(1-D)** $\Delta x$ = Area[$v(t)$]

Average Acceleration: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ **(1-D)** *"Instantaneous"* = *"slope"* $\Delta v$ = Area[$a(t)$]

## Constant Acceleration Equations for 1-D Motion

$$x = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2 \qquad v = v_{ox} + a_x t \qquad v_x^2 = v_{ox}^2 + 2a_x(x - x_o) \qquad v_{ave} = (v_0 + v_f)/2$$

**For the free fall (y-axis is *UP*; *g* ≈ 10 m/s²):**

$$v_y = v_{yi} - gt \qquad y_f = y_i + v_i t - \tfrac{1}{2}10t^2 \qquad v_y^2 = v_{oy}^2 - 2g(y - y_o) \qquad v_{y\,ave} = (v_{y\,0} + v_{y\,f})/2$$

**For the projectile motion ($y$ - axis is UP; $g \approx 10$ m/s$^2$):** $v_x = v_{x0} = const$

$x = x_0 + v_x t$ $v_y = v_{iy} - gt$ $y = y_o + v_{oy} t - ½ g t^2$ $v_i = \sqrt[2]{v_{ix}^2 + v_{iy}^2}$

$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$

## Newton's Laws

$\vec{F}_{Net}$ **definition:** $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \ldots$ **Translational Equilibrium:** $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \ldots = 0$ **and** $\boldsymbol{v} = 0$

Newton's Second Law: $\boldsymbol{F}_{Net} = \Sigma \boldsymbol{F} = m\boldsymbol{a}$ Newton's Third Law: $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$

Friction: *kinetic* $F_{k\,fr} = \mu_k * N$ *static* $F_{s\,fr} \le \mu_s * N$ $F_{s\,fr\ max} = \mu_s * N$

Weight: $\boldsymbol{W} = m\boldsymbol{g}$ Apparent weight: $|AW| = |N|$ *or* $|AW| = |T|$

## Energy and Work:

Work (constant force): $W = |F||S| \cos\theta$ ($\theta$ is the angle between force and displacement)

Kinetic Energy: $K = K_{tr} = ½ mv^2$

Work done by friction: $W = - F_{fr} L$ ($L$ is the distance traveled)

Power: $P = \dfrac{W}{t} = |F||v| \cos\theta$

Gravitational Potential Energy ($y$-axis is *UP*): $U_G = mgy$ Elastic Potential energy: $U_E = ½ kx^2$

Energy Conservation:

Master equation: $U_i + K_i + W_{nc} = U_f + K_f$ where $W_{nc}$ is work done by non-conservative forces, including force of friction.

Work – Kinetic Energy theorem: $K_f - K_i = W_{total} = W_{net} = W_1 + W_2 + \ldots$

## Systems of Particles + Momentum:

Center of mass: $X_{CoM} = \dfrac{x_1 m_1 + x_2 m_2 + \ldots}{m_1 + m_2 + ..}$ (similar expression is for $Y_{CoM}$)

Momentum: $\boldsymbol{p} = m\boldsymbol{v}$ Impulse: $\boldsymbol{J} = \Delta \boldsymbol{p} = \boldsymbol{F}_{ave} \Delta t$ = Area $\{F(t)\}$

Linear Momentum conservation ($\boldsymbol{F}_{\text{Net System}} = 0$ or $\Delta t \approx 0$): $\vec{p}_{system\ initial} = \vec{p}_{system\ final}$

$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 + \ldots$

Elastic collision ($KE$ = const, all momenta point "to the right"): $v_1 + u_1 = u_2 + v_2$

## Rotation:

$\theta = \dfrac{s}{r}$ $\omega = \dfrac{v}{r}$ $\alpha = \dfrac{a_t}{r}$ $N = \dfrac{S}{2\pi r} = \dfrac{\theta_{(in\,rad)}}{2\pi} = \dfrac{\theta_{(in\deg)}}{360}$

$T = t/N = 1/n$ $n = f = 1/T$ $\omega T = 2\pi$

Centripetal acceleration: $a_c = \dfrac{v^2}{R}$ ; total acceleration: $a_{total} = \sqrt{a_c^2 + a_t^2}$

Constant angular acceleration equations: $\omega = \omega_o + \alpha t$ $\theta = \theta_o + \omega_o t + ½ \alpha t^2$

Torque : $|\tau| = |rF\sin\theta| = |F|l$ ($l$ – the shortest distance from the axis of rotation to the line of action of the force)

Equilibrium : $\sum \vec{F} = 0$ ***and*** $\sum \vec{\tau} = 0$ CCW = "positive" CW = "negative

Rotational dynamics: $\vec{\tau}_{net} = I\vec{\alpha}$ $I_{disk} = \frac{1}{2}MR^2$ $I_{sphere} = \frac{2}{5}MR^2$ $I_{rod} = \frac{1}{12}ML^2$ $I_{ring} = MR^2$

Rotational Kinetic Energy: $K_{rot} = ½I\omega^2$

Angular momentum: $L = I\omega$ (for a solid object) $|L| = |rp\sin\theta|$ (for a point mass)

Angular Momentum conservation: $\vec{L}_{system\ initial} = \vec{L}_{system\ final}$

**Rolling (no slipping, no non-conservative forces):** $\omega = \dfrac{v_{CofM}}{r}$ $\alpha = \dfrac{a_{CofM}}{r}$

$U_i + K_i = U_f + K_f$ $K_{total} = K_{tr} + K_{rot}$

$K_{rot} = ½I\omega^2$ $K_{tr} = ½mv^2$

## SHM

$|F_{el}| = k|\Delta l| = k|x|$ $U_{el} = ½k(\Delta l)^2 = ½kx^2$ $a_x = -\omega^2 x$ $x = A\cos(\omega t + \theta_0) + D$

$v = -A\omega\sin(\omega t + \theta_0)$ $a = -A\omega^2\cos(\omega t + \theta_0)$

$\omega T = 2\pi$ $v_{max} = A\omega$ $a_{max} = A\omega^2$ $f = 1/T$ (spring) $\omega^2 = \dfrac{k}{m}$

Pendulum, small angles: $\omega^2_{simple} = \dfrac{g}{l}$ (physical pendulum) $\omega^2_{physical} = \dfrac{g}{l_{eff}} = \dfrac{mgL}{I}$

**Fluids:**

Pressure: $P = \dfrac{F}{A}$ Buoyant Force: $F_b = \rho_{fluid} V_{disp} g$ $\rho_{water} = 1$ gram/cm$^3$

Density: $\rho = M/V$ (static liquid) $P = P_{Atm} + \rho gh$ 1 Atm $\approx 10^5$ Pa

Continuity: $A_1 v_1 = A_2 v_2$ VFR = $Av$ = const MFR = $\rho$*VFR = $\rho Av$

Bernoulli's Equation: $\rho g y_1 + ½\rho v_1^2 + P_1 = \rho g y_2 + ½\rho v_2^2 + P_2$

**Gravity:** $|F| = G\frac{m_1 m_2}{r^2}$ $U = -G\frac{m_1 m_2}{r}$ Kepler's law (3[d], circular orbits): $\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2$ or

$\frac{4\pi^2}{T^2} = \frac{GM}{r^3}$ $G \approx 7\times10^{-11}$ (SI units).

## Temperature and Heat; Gases and Kinetic Theory

$Q = \Delta U_{\text{int}} + W_{\text{by system}}$ $Q = C\Delta T$ $C = mc$ $Q_{solids} = Q_{liquids} = mc\Delta T$ $Q_{\text{gases}} = nc_{molar}\Delta T$

$Q_{\text{phase\_change}} = \pm Lm$ isolated/insulated system: $\sum Q = 0$ $T(\text{K}) \approx t\,(\text{C}) + 273$

For solids: linear expansion $\Delta L = \alpha L_o \Delta T$ volumetric expansion $\Delta V = 3\alpha V_o \Delta T$

**Ideal Gas:** $\text{KE}_{1\_tr\_\text{ave}} = \frac{1}{2}m_0 v^2_{\text{rms}} = \frac{3}{2}kT$ $\text{KE}_{1\_tr+rot\_\text{ave}} = \frac{i}{2}kT$ $PV = nRT = NkT$

$U_{\text{int}} = (i/2)nRT = (i/2)PV$ $n = N/N_A = m/M$ $M = N_A * m_0 = m/n$

$W_{\text{cycle}} = \sum W_{\text{process}} = Q_{\text{cycle}} = \sum Q_{\text{process}} = Q_{\text{h}} - |Q_{\text{c}}|$ $\Delta U_{T=\text{const}} = 0$ $\Delta U_{\text{cycle}} = \sum \Delta U_{\text{process}} = 0$

$W_{\text{process}}$ = *area under the P-V curve* $W_{p=const} = P\Delta V$ $W_{\text{isotherm}} = P_1 V_1 \ln\frac{V_2}{V_1}$

$e = \frac{Q_h - |Q_c|}{Q_h} = \frac{W_{cycle}}{Q_h}$ $e_{\text{ideal}} = \frac{T_h - T_c}{T_h}$ $\Delta S = Q/T$ ($T=const$) $\Delta S = Q/T_{\text{Ave}}$ (small $\Delta T$)

$R = kN_{\text{A}} \approx 8$ J/(K mole) $k \approx 1.4\text{x}10^{-23}$ J/K $N_A \approx 6\text{x}10^{23}$ mole$^{-1}$ 1 cal ≈ 4 J